Abstract: This paper presents the design of a $H_\infty$ controller dedicated to the lateral driving assistance including lane keeping and yaw dynamic enhancement. It is used to add a steering torque to that of the driver in order to improve the rejection of external disturbances (such as lateral wind) despite uncertainties such as road adhesion, longitudinal velocity, and driver dynamics.

The simulation results presented in the paper demonstrate the effectiveness of the proposed solution. Using non linear model of the vehicle, lane keeping and lane change maneuvers are made more accurate, while the external disturbances are well rejected. Vehicle trajectories are simulated with respect to various road adhesion, curvature and speed, in order to evaluate vehicle stability domain and kind of instability that may occur.

Keywords: Robust control, $H_\infty$ optimization, Uncertain linear systems.

1. INTRODUCTION

Lateral control finds many applications in the field of driving assistance. A large number of vehicle accidents results from unexpected excessive yaw motion such as spin-out and lane departure. In addition, such type of accidents generally occurs on rural road, and about 30% of fatalities in France are due to accidents with vehicle alone.

In a close future, the steer-by-wire systems will be in great deployment. The study led in this paper presents an intermediary solution which consists in adding a supplementary torque using a motorized direction system to that of the driver, in situations of lane keeping or lane departure avoidance and ensuring the driving comfort criteria. (Ackermann et al., 1999), (J.-C. Hsu and Yeh, 1998), (Mammar and Koenig, 2002).

This work is developed in the field of a large french project named ARCoS (Action on safe driving) which aims at improving road safety using active correcting device.

Ground vehicles are complex systems subject to many parameter variations such as mass, speed, adhesion, uncertain dynamics and disturbances such as lateral winds and road banking. Thus Robust control methods such as $H_\infty$ control are suitable (J.C. Doyle and Francis, 1989), (Duc and Font, 1999), (McFarlane and Glover, 1990).

This paper presents first the model used for the synthesis strategy in section 2. In section 3, control objectives are given. Next in section 4, control system design is developed using $H_\infty$ optimal con-
controtheory. Simulation results are provided in section 5. This section is dedicated to the controller analysis respect to various driving maneuvers and conditions.

2. SYNTHESIS MODEL DESIGN

The synthesis model is composed of 3 parts which describe the vehicle, the steering column and the driver respectively.

Concerning the vehicle, the classical fourth order linear model is used (J. Ackermann and Steinhauser, 1993), (Chee and Tomizuka, 1994). The state vector is $x_v = [\beta, r, \psi_L, y_L]^T$. Its components are the vehicle side slip angle ($\beta$), the yaw rate ($r$), the relative yaw angle ($\psi_L$) and the lateral offset ($y_L$) from the centerline. The state space model is the following:

$$
\begin{align*}
\dot{x}_v &= A.x_v + B.e_v \\
z_v &= C.x_v + D.e_v
\end{align*}
$$

where

$$
A = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 \\
a_{21} & a_{22} & 0 & 0 \\
0 & 1 & 0 & 0 \\
v & l_S & v & 0 \end{pmatrix},
B = \begin{pmatrix} b_1 & e_{11} & 0 \\
b_2 & e_{22} & 0 \\
0 & 0 & 0 \\
v & 0 & 0 \end{pmatrix}
$$

$$
C = \begin{pmatrix} 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-T_{S\beta} & -T_{Sr} & 0 & 0 \\
0 & 0 & 0 & 0 \end{pmatrix}
$$

$$
D = \begin{pmatrix} 0 & 0 & 0 \\
2K_p\psi_L \eta & 0 & 0 \\
\frac{R_S}{K_p}\psi_L \eta & 0 & 0 \end{pmatrix}
$$

The input vector is $e_v = [\delta_f, f_w, \rho_{ref}]^T$, where $\delta_f$ is the steering angle of the front wheels in rad.s$^{-1}$, $f_w$ is the disturbance wind force in Newton and $\rho_{ref}$ is the road curvature in m$^{-1}$. The output vector is $z_v = [\psi_L, y_L, T_S]^T$. The last output is the front wheel aligning torque $T_s$. The parameters are given below and are presented in the nomenclature at the end of the paper.

$$
\begin{align*}
a_{11} &= -\frac{2(c_r + c_f)}{m v J} & a_{12} &= -1 + \frac{2(l_r c_r - l_f c_f)}{j v} \\
a_{21} &= \frac{2(l_r c_r - l_f c_f)}{J} & a_{22} &= \frac{2(l^2_r c_r + l^2_f c_f)}{j v} \\
c_r &= c_{0_r} \nu & c_f &= c_{0_f} \nu \\
b_1 &= \frac{2c_f}{m v l} & b_2 &= \frac{2c_f}{l} \\
e_{11} &= \frac{m v}{2K_p c_f \eta \chi} & e_{22} &= \frac{v}{J} \\
T_{S\beta} &= \frac{2K_p c_f \eta \chi}{R_S} & T_{Sr} &= \frac{J}{2K_p c_f \eta \chi} 
\end{align*}
$$

An Electronic Power Steering (EPS) mechanism models the steering column (Figure 1). The inputs are the electric motor torque $T_a$, the driver steering torque $T_d$ and the front wheels aligning torque $T_s$. The output $\delta_f$ is the steering angle of the front wheels which is an input of the vehicle model.

Finally, the synthesis model is completed with a driver model for lane keeping maneuver (Figure 2). This contains several components (Modjtabaisadeh and Hess, 1993). The first one is called structural model. It is constituted by a time delay representing inherent human processing time and neuro-motor dynamics. This component represents the high frequency driver compensation component and is modelled by a dead time $T_D = 0.2s$. In order to keep a state space formulation, this time delay is approximated here by a first order Padé representation.

A second order low pass filter with damping factor $\xi = 0.707$ and natural frequency $\omega_n = 10$rad.s$^{-1}$ allows to structure the driver model.

It must be noted that all these parameters are not constant and are only valid for restricted vehicle and driver configurations (Day and Metz, 2000). This model is fed by the driver perception of the driving environment which is reduced in this case of lane keeping tasks to lateral displacement $y_L$ and the relative yaw angle $\psi_L$. A more complex perception system is needed when obstacle detection and avoidance is included in the task. The driver model presents a third input represented by the torque applied by the assistance $T_a$ which is sensed by a proprioceptor in muscle tissue and processed reflexively in the spinal cord level.

The controller synthesis model is then obtained by embedding the driver, the steering column and the vehicle models. The control input is the assistance torque $T_a$.

It is assumed that $y_L$ and $\psi_L$ are the only measurements which are available for feedback. Figure 3 presents the control strategy so obtained. Disturbances are mainly the lateral wind which is applied to the vehicle and the control input is the steering torque $T_a$.

In part 5, a non-linear model of the vehicle is integrated in the closed loop strategy in order to analyze and validate the control performances.
3. CONTROL OBJECTIVES

The lateral assistance strategy must operate in the cases of both lane keeping and lane change maneuvers, in straight and curved road sections. According to the model of section 2, during car displacement, the vehicle is under wind force disturbances. The controller has to minimize the effects of the external input \( f_w \) for example, on the outputs \( y_L \) and \( \psi_L \), which should be close to zero with limiting actuator effort. The following requirements must be ensured:

- lateral displacement at the gravity center of the vehicle from the lane center < 20 cm.
- relative yaw angle < 0.01 radian.
- lateral acceleration < 0.2g (1.92 m.s\(^{-2}\)) for comfort and safety of the passengers.
- \( T_a < 10 \) N.m.

Moreover, some parameters are subject to uncertainties (adhesion, driver features) (Mammar, 2001), (Mammar and Koenig, 2002) and others vary such as longitudinal velocity. Thus the controller must ensure the robust stabilization of the closed loop system under these parameter and dynamic variations.

4. CONTROL SYSTEM DESIGN

The design procedure adopted in this paper is based on \( H_\infty \) control theory. The control input takes the following form:

\[
T_a(s) = K(s).Z(s)
\]

Where \( z = [y_L, \psi_L]^T \) is the measurement vector used for feedback (capital letters are used for Laplace transforms).

The control requirements are met using weighting frequency-dependent filters \( W_i \). They are introduced to weight the control and the outputs in order to improve the stability margin of the system and to ensure good closed-loop performance in the sense of classical Automatic control.

Figure 4 presents the control system design. The signals to be regulated are \( y_L^*, \psi_L^* \) and \( T_a^* \) face to the disturbance \( f_w^* \). The controller is fed by the measurements \( y_L \) and \( \psi_L \). The weighting functions \( W_i, \ i = 1..5 \) are first order filters, parametrized by static gain \( G_0 \), gain at infinity \( G_\infty \) and frequency \( \omega_0 \), whose expression is the following:

\[
W_i(s) = \frac{G_\infty \sqrt{|G_0^2 - 1|s + G_0\omega_0 \sqrt{|G_\infty^2 - 1|}}}{\sqrt{|G_0^2 - 1|s + \omega_0 \sqrt{|G_\infty^2 - 1|}}}
\]

The gain of \( W_1 \) is high at low frequency in order to efficiently control the lateral displacement \( y_L \). The weighting function \( W_2 \) restricts simultaneously \( \psi_L \). \( W_3 \) stands for a approximative disturbance model, and emphasizes low frequencies. \( W_4 \) weights the control input \( T_a \) and adds constraints at low frequency (in this range, the driver must control the vehicle alone). \( W_5 \) is constant and weights the 2 measurement contributions (if \( W_5 \) is high, the controller doesn’t take into account \( \psi_L \)). The parameters are finally chosen as follows:

<table>
<thead>
<tr>
<th>Filter</th>
<th>( G_0 )</th>
<th>( G_\infty )</th>
<th>( \omega_0 ) (rad.s(^{-1}))</th>
</tr>
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<tbody>
<tr>
<td>( W_1 )</td>
<td>300</td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>( W_2 )</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>( W_3 )</td>
<td>20</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>( W_4 )</td>
<td>30</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>( W_5 )</td>
<td>0.1</td>
<td>0.1</td>
<td>5</td>
</tr>
</tbody>
</table>

Then, the feedback controller \( K(s) \) is synthesized using a \( \gamma \)-iteration algorithm in order to minimize the transfer between the normalized disturbance \( f_w^* \) and the outputs \( y_L^*, \psi_L^* \) and \( T_a^* \) in the sense of \( H_\infty \) norm:

\[
\min \| T[y_L^*, \psi_L^*, T_a^*] - f_w^* \|_\infty
\]

Considering that longitudinal velocity of the vehicle is \( v = 10 \) m.s\(^{-1}\) and road condition is \( v = 0.8 \), the algorithm leads to \( \gamma_{\text{opt}} = 2.70 \) for the minimal value of the \( H_\infty \) norm (7).

The controller degree is 12. The Black diagram of the controlled open-loop is shown on Figure 5. The plot exhibits good phase and gain margins. One can then expect good robustness properties. Following an order reduction based on Hankel singular values truncation which leads to a 4 order controller, the assistance strategy is associated to the nonlinear model in order to perform the
simulations. Finally, the controller is integrated in the closed loop and performs lateral driving assistance in interaction with the driver.

In order to appreciate the assisted vehicle behavior face to disturbances, the next section presents simulation results.

5. SIMULATION RESULTS

This section proposes to disturb the assistance system and to analyze the responses. The controller which is synthesized for the operating point \( v = 10 \text{m.s}^{-1}, \nu = 0.8 \) is then integrated to the closed loop as shown on Figure 3.

The assistance strategy is tested in various situations such as lateral wind force disturbances, road adhesion loss and speed variations. Consequently, a nonlinear vehicle model which is detailed in the following section is integrated.

5.1 Tire-road nonlinear model

As we are only concerned with lateral control, a simple nonlinear model of a vehicle is derived by neglecting heave, roll and pitch motions. This model includes the two translational motions and the yaw motion. The vehicle wheels are numbered from 1 to 4. The interaction between the tire \( i \) \((i = 1, 2, 3, 4)\) and the road surface is decomposed into longitudinal forces \( f_x \) \((\lambda_i)\) and lateral forces \( f_y \) \((\alpha_i)\). These forces will be detailed below. The nonlinear model is obtained by writing the translational and rotational equations in the vehicle fixed frame where \( v_x \) and \( v_y \) represent the velocity:

\[
\begin{align*}
m(\dot{v}_x - v_yr) &= f_{x1} + f_{x2} - f_{y1} + f_{y2} \\
m(\dot{v}_y + v_xr) &= f_{x3} + f_{x4} - f_{y3} + f_{y4} \frac{\Delta f_y}{f_y} \cos \delta_f \\
J\dot{\gamma} &= \Delta f_x - \Delta f_y \sin \delta_f + l_wf_w
\end{align*}
\]

where

\[
\begin{align*}
f_{x1} &= f_{x3} + f_{x2} + f_{y1} + f_{y2} \\
f_{x2} &= f_{x4} + f_{x3} + f_{y3} + f_{y4} \\
\Delta f_x &= (f_{x3} - f_{x2}) + (f_{x2} - f_{x1}) \cos \delta_f \\
\Delta f_y &= f_{y2} - f_{y1}
\end{align*}
\]

Table 1. Nonlinear formulas of tire slip angles

<table>
<thead>
<tr>
<th>(\alpha_1)</th>
<th>(\alpha_2)</th>
<th>(\alpha_3)</th>
<th>(\alpha_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta_f - \tan^{-1} \left( \frac{v_y - (n_x \cos \delta_f) \delta_f - (n_x \cos \delta_f - l_x)v_t}{v_x + (n_x \sin \delta_f) \delta_f + (n_x \sin \delta_f - \frac{\Delta f_y}{f_y})v_t} \right))</td>
<td>(\delta_f - \tan^{-1} \left( \frac{v_y - (n_x \cos \delta_f) \delta_f - (n_x \cos \delta_f - l_x)v_t}{v_x + (n_x \sin \delta_f) \delta_f + (n_x \sin \delta_f - \frac{\Delta f_y}{f_y})v_t} \right))</td>
<td>(-\tan^{-1} \left( \frac{v_y - l_x v_t}{v_x + \frac{\Delta f_y}{f_y}} \right))</td>
<td>(-\tan^{-1} \left( \frac{v_y - l_x v_t}{v_x + \frac{\Delta f_y}{f_y}} \right))</td>
</tr>
</tbody>
</table>

The longitudinal forces depend directly on the tire slip coefficient \((\lambda_i)\) while the lateral forces depend on the tire slip angles \((\alpha_i)\). Table 1 summarizes the expressions of the tire slip angles. In this paper, the magic formula of Pacejka (Pacejka, 1979) is used for each tire in order to determine the lateral forces:

\[
f_{yi}(\alpha_i) = d_i \sin[c_i \tan^{-1} \left( \frac{b_i(1 - e_i) \alpha_i + e_i \tan^{-1}(b_i \alpha_i)}{e_i} \right) ] \tag{9}
\]

The coefficients \(b_i, c_i, d_i, e_i\) depend on the tire characteristics, on the road conditions, and on the vehicle operational conditions.

5.2 Disturbance rejection

A 500N lateral wind force disturbance, presented by Figure 6, is applied to the vehicle which is moving on a straight road. The vehicle is now supposed to be at velocity \( v = 20 \text{m.s}^{-1} \) and road adhesion \( \nu = 0.8 \). On Figure 7, the solid lines represent the responses of the assisted vehicle and the

![Figure 5. Black diagram for the controlled open-loop behavior face to disturbances, the next section presents simulation results.](image)

![Figure 6. Lateral wind force time in second](image)

![Figure 7. (a) \(y_L\) with a 500N lateral gusts of wind (b) relative yaw angle](image)
dotted lines stand for the not assisted ones. One can note from Figure 7 (a) in solid line, that $y_L$ is greatly reduced and thus the controlled vehicle remains closer to road centerline. The disturbance is practically rejected. The maximum value is under 20cm which is the acceptable maximum deviation. Figure 7 (b) similarly presents the relative yaw angle which stays under 0.01 rad as required. Furthermore, the assistance torque is less than 3N.m as shown on Figure 8 (a), which is compatible with the actual EPS actuator performances. Figure 8 (b) presents the lateral acceleration which is significantly under 0.2g as required. Consequently, one can conclude on the remarkable effectiveness of the synthesized $H_\infty$ controller.

![Image](image1.png)

**Fig. 8.** (a) Assistance torque $T_a$ (b) Lateral acceleration

### 5.3 Lane change maneuver

The handling improvement is now investigated in case of lane change maneuver. The vehicle is supposed to be at 10 $m.s^{-1}$ and $\mu = 0.8$ and enters in a typical chicane which is presented on Figure 9. In this situation, on Figure 10 (a), $y_L$ at the gravity center with the lateral control assistance in solid line is compared to the not assisted one in dotted line. Considering the abilities of the driver, the result is thus satisfying. Moreover, lateral acceleration presented on Figure 10 (b), respects the specifications.

![Image](image2.png)

**Fig. 9.** Typical chicane

![Image](image3.png)

**Fig. 10.** (a) Comparison of the lateral displacements with and without assistance (b) Lateral acceleration

In order to reject the disturbances. The second, in dotted line, presents the trajectory of the vehicle assisted by the control system excepted in the situation of lateral wind disturbances and adhesion loss.

![Image](image4.png)

**Fig. 11.** Test track presentation

Figure 12 presents the lateral offsets at the vehicle center of gravity. The vehicle is able to progress depending on the road curvature under disturbances. The performances of the assisted vehicle in solid line, are better than the not assisted one in dotted line. Nevertheless, the lateral offset at the vehicle center of gravity is not inferior to 20 cm on the whole track. In fact, overshoots occur in small curvature areas of the road which correspond to difficult turns. However, the $H_\infty$ assistance strategy is particularly efficient.

### 5.5 Aquaplaning maneuver

In this case the vehicle is considered to perform a stationary cornering maneuver at 20 $m.s^{-1}$. The driver steering angle is then constant. It is also assumed that at $t = 10, 15, 20, 25$ sec, the tires cross a low friction region of the road which...
corresponds to adhesion \( \mu = 0.1 \). The tires returns to a dry road 2 seconds later. The lateral control assistance is active and results are presented. Figure 13 (a) exhibits the nominal trajectory of the vehicle and sliding road sections where the vehicle is subject to aquaplaning. Figure 13 (b) shows the lateral offset at \( l_S \), in solid line, while the vehicle is assisted. It deviates less than 20 cm from its nominal path as required in the control objectives. \( y_{ls} \) without the controller, in dotted line, shows that the vehicle is out the road and the importance of the assistance. Finally, the closed loop system is stable, and the \( H_\infty \) controller is proved to be particularly efficient.

![Diagram](image-url)

Fig. 13. (a) Nominal vehicle trajectory
(b) Comparison of the lateral displacements at \( l_S \) with and without assistance

6. CONCLUSION

In this paper, \( H_\infty \) optimization applied to lateral driving assistance has been investigated. The obtained controller has robust properties face to input disturbances such as lateral wind force and parameter variations. The paper also presents several simulation results in various situations, disturbance rejection, lane change maneuver, test based on experimental data and aquaplaning maneuver. Considering that the \( H_\infty \) synthesis is related to the operating point \((v,v)\), the efficiency of the controller is established and results are promising. Finally, the control strategy will be tested on an experimental vehicle dedicated to lateral driving assistance. Moreover, in order to improve the velocity domain under the required performances, gain scheduling with respect to velocity is the future work.

REFERENCES


Mammar, S. (2001). Feedforward and feedback control for vehicle handling improvement by active steering. 3rd IFAC Workshop on Advances in Automotive Control pp. 135–140.


Nomenclature

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_S ) steering system damping coefficient</td>
<td>10</td>
</tr>
<tr>
<td>( c_{fa} ) front cornering stiffness</td>
<td>40000 N.rad(^{-1})</td>
</tr>
<tr>
<td>( c_{ra} ) rear cornering stiffness</td>
<td>35000 N.rad(^{-1})</td>
</tr>
<tr>
<td>( l_S ) inertial moment of steering system</td>
<td>1kg.m(^2)</td>
</tr>
<tr>
<td>( J ) vehicle yaw moment of inertia</td>
<td>2454 kg.m(^2)</td>
</tr>
<tr>
<td>( K_F ) proportional gain</td>
<td>0.1</td>
</tr>
<tr>
<td>( K_L ) proportional gain</td>
<td>20</td>
</tr>
<tr>
<td>( K_p ) manual steering</td>
<td>1</td>
</tr>
<tr>
<td>( l_f ) distance from CG to front axle</td>
<td>1.05m</td>
</tr>
<tr>
<td>( l_r ) distance from CG to rear axle</td>
<td>1.56m</td>
</tr>
<tr>
<td>( l_S ) look-ahead distance</td>
<td>5m</td>
</tr>
<tr>
<td>( l_m ) distance ahead CG</td>
<td>0.4m</td>
</tr>
<tr>
<td>( m ) total mass</td>
<td>1500 kg</td>
</tr>
<tr>
<td>( R_S ) steering gear ratio</td>
<td>21</td>
</tr>
<tr>
<td>( s_d ) driving axle length</td>
<td>1.5 m</td>
</tr>
<tr>
<td>( v ) longitudinal velocity</td>
<td>([5,25]).m.s(^{-1})</td>
</tr>
<tr>
<td>( n_t ) tire length contact</td>
<td>0.13m</td>
</tr>
<tr>
<td>( \nu ) adhesion</td>
<td>([0.1])</td>
</tr>
<tr>
<td>( \xi ) damping factor</td>
<td>0.707</td>
</tr>
<tr>
<td>( \tau_L ) driver mental load</td>
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</tr>
<tr>
<td>( \tau_p ) dead time</td>
<td>0.151s</td>
</tr>
<tr>
<td>( \omega_n ) natural frequency</td>
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