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Two-degree-of-freedom Formulation of Vehicle Handling Improvement by Active Steering

Saïd Mammar † and Vahé Badal Baghdassarian ‡
† CEMIF, Université d'Evry 91025, Evry Cedex, France
mammar@iup.univ-evry.fr
‡ LIVIC, Institut National de Recherche sur les Transports et leur Sécurité
13, route de la Minière, 78000 Versailles, France

Abstract

This paper presents coprime factors based two-degree-of freedom H_∞ control for vehicle handling improvement. Control synthesis uses a linear vehicle model which includes the yaw motion and disturbance input. The synthesis procedure allows the separate processing of the robust stabilization problem and reference signal or disturbance rejection. The control action is applied as an additional steering angle, by combination of the driver input and feedback of the yaw rate. The synthesized controller is tested in both disturbance rejection and driver imposed yaw reference tracking maneuvers. The maneuvers are completed at different speeds and road conditions.

1 Introduction

Vehicle handling improvement has been investigated for both 4 wheel steering and 2 wheel steering cars [4][1]. In [1] an analytical method which allows robust unilateral decoupling of the yaw rate from the lateral dynamics has been presented. The controller output consists in an additional steering angle obtained by integration of the difference between the reference yaw rate value as commanded by the driver and the actual achieved vehicle yaw rate. Several refinement have been introduced in [2] in order to improve vehicle handling while ensuring similar steady state behavior for both the controlled and the conventional car.

In this paper a two-degree-of-freedom vehicle handling scheme is developed. It allows robust model matching against parameters variations and rejection of lateral forces and torque disturbances which may rise from wind forces. The controller configuration also allows a simple and a direct implementation on actual electrically steered vehicles.

The paper is organized as follow: section 2 introduces the vehicle model which is used for active steering controller synthesis. Controller objectives and synthesis methodology are presented in section 3. Controller implementation and several simulation results are given in section 4.

2 Vehicle lateral model

The model used for control synthesis is derived from the bicycle model in which all the angles are assumed to be small [3]. The lateral translation and rotational yaw motion equations written in the vehicle fixed frame take the following form

$$\begin{bmatrix} mv(\dot{\beta}+r) \\ J\dot{r} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ l_f & -l_r & l_w \end{bmatrix} \begin{bmatrix} f_f(\alpha_f) \\ f_r(\alpha_r) \\ f_w \end{bmatrix} \quad (1)$$

where β is the vehicle side slip angle, r is the yaw rate and f_w is a wind force acting at the aerodynamical center of the side surface at a distance l_w from the center of gravity. m is the vehicle mass while J is the vehicle moment of inertia. Furthermore, it is assumed that tire side slip angles, $\alpha_f = (\delta_f - \beta_f)$ and $\alpha_r = -\beta_r$ are also small and thus the lateral forces $f_f(\alpha_f)$ and $f_r(\alpha_r)$ linearly depend on these variables

$$f_f(\alpha_f) = c_f\alpha_f = \mu c_f^* \left(\delta_f - \beta - \frac{l_f}{v}r\right) \quad (2)$$

$$f_r(\alpha_r) = c_r\alpha_r = -\mu c_r^* \left(\beta - \frac{l_r}{v}r\right) \quad (3)$$

Parameters c_f and c_r are respectively the front and rear tire cornering stiffness. The common road adhesion coefficient μ is introduced by considering equal variations of the front and rear tire cornering stiffness. In the previous equations c_f and c_r are changed to $(c_f = \mu c_f^*)$ and $(c_r = \mu c_r^*)$. The parameter μ varies in the range $[0, 1]$ which reflects different road adhesion conditions. c_f^* and c_r^* are thus respectively the nominal front and rear tire cornering stiffness

The following numerical values are used in simulation, they correspond to an understeering medium class eu-

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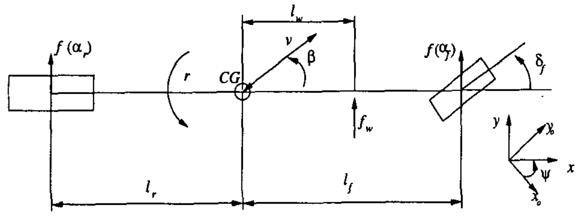

Figure 1: Lateral model data of the vehicle

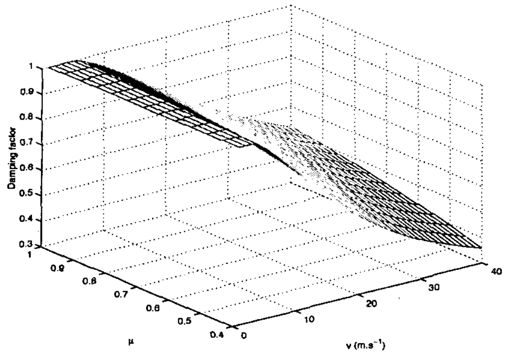

Figure 2: Damping factor function of speed and road adhesion

ropean vehicle.

$c_f^* = 49400 N.rad^{-1}$	$m = 991 Kg$	$l_f = 1.00m$
$c_r^* = 54400 N.rad^{-1}$	$J = 1574 Kg.m^2$	$l_r = 1.46m$
	$l_w = 0.4m$	

The nominal forward speed for controller synthesis is chosen as $v = 20 m.s^{-1}$. However v is considered to be a varying parameters in the range $[1, 40]\, m.s^{-1}$.

This model takes the form

$$\dot{x} = Ax + B_w w + B_u u \tag{4}$$

where $x = [\beta, r]^T, w = f_w$ is the disturbance input, $u = \delta_f$ is the steering angle.

Speed and road adhesion variations affect both the transient and the steady state behavior of the vehicle. Steady state side slip angle value changes sign below a certain speed $v_\beta = \sqrt{\frac{l_r(l_r+l_f)c_r}{m l_f}}$. Increasing speed and road adhesion reduction have the same effect and lead to damping reduction (figure 2).

3 Control objectives and design methodology

The control objectives are primarily twofold:

- steady state rejection of step input disturbances such side wind force or yaw torque disturbance

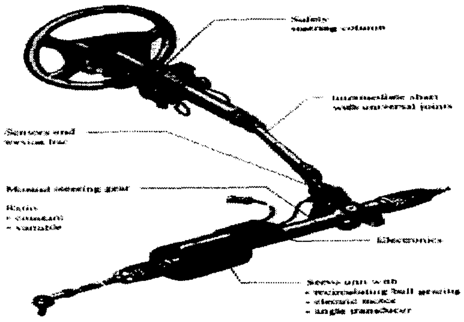

Figure 3: Electrical steering actuator

- enhancement of vehicle responses damping at all operating speeds while ensuring practically identical transient behavior for all speeds and road adhesion. The vehicle has also to present similar steady state behavior as the conventional car without active steering.

The above two points summarizes the needed vehicle handling improvement.

3.1 Two-degree-of-freedom formulation

Considering vehicle active steering, the tire steering angle δ_f is set in part by the driver through the vehicle classical steering mechanism while an additional steering angle δ_c is set by the controller (figure 3)

$$\delta_f = \frac{\delta_d}{R} + \delta_c \tag{5}$$

$$\quad = \delta_w + \delta_c \tag{6}$$

where δ_d is the steering wheel angle set by the driver. and R is the gear ratio ($R = 21$).

Let G be the transfer function from the steering angle δ_c to the yaw rate r. In the sequel, the dependence on s is omitted but it will be clear from the context.

$$r = G\delta_c = \left(\tilde{M}^{-1}\tilde{N}\right)\delta_c \tag{7}$$

where $(\tilde{M}, \tilde{N})$ are normalized left coprime factors. In the following, It is assumed that speed and road adhesion variations are modeled as additive uncertainties on normalized left coprime factors. This leads to a set of perturbed plants G_Δ[8].

The first control objective is achieved by introducing an integral action upstream the disturbance input. This is acheived by introducing a weighting compensator of the form of a PI filter according to the loop shaping design methodology [8].

$$W(s) = \frac{0.1s + 1}{s} \tag{8}$$

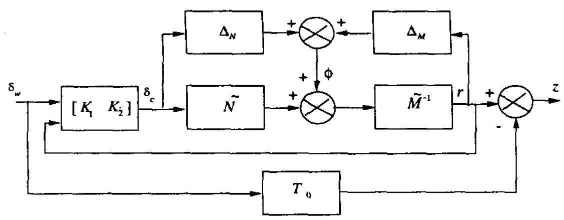

Figure 4: Two-degree-of-freedom configuration

Let now G_s be the shaped plant ($G_s = WG$), using normalized left coprime description, one can write

$$G_s = \tilde{M}_s^{-1}\tilde{N}_s \tag{9}$$

Damping enhancement may be achieved by feedback control while the best candidate for identical transient behavior is robust model matching which is taken into account in two-degree-of-freedom control.

Model matching is used when it is necessary to specify some time domain objectives such as overshot of step response and setting time. These time domain objectives are grouped into the transfer function, which may be considered as the desired closed loop transfer function. The two-degree-of-freedom configuration of a controller permits the separate processing of the reference and measurement signals. Let now T_0 be the desired transfer function between δ_w and r (figure 4). According to the configuration of figure 6, the controller can be split into two parts $K_1$and K_2[6][9]. The control input is then given by

$$\delta_c = K_1\delta_w + K_2 r \tag{10}$$

K_1 acts as a prefilter of the reference signal while the feedback controller K_2 ensures robust stability and damping between the reference signal δ_w and the output r (figure 4). In addition to the internal robust stability of the closed loop system of figure 3, the controller has to keep the error signal z small for an entire family of perturbed plants G_Δ. This fact constitutes robust model matching which corresponds to $\left\|(I - G_pK_2)^{-1}G_pK_1 - T_0\right\|_\infty \le \gamma$ for all perturbed plant models G_p in G_Δ.

Let the two-degree-of-freedom controller be denoted by $K = [K_1, K_2]$. The transfer matrix M from $[\delta_w, \phi]^T$ to $[z, r, \delta_c]^T$ is given by

$$M = \left[\begin{array}{c|c} (1-G_sK_2)^{-1}G_sK_1 - T_0 & (1-G_sK_2)^{-1}\tilde{M}_s^{-1} \\ \hline G_s(1-K_2G_s)^{-1}K_1 & (1-G_sK_2)^{-1}\tilde{M}_s^{-1} \\ (1-K_2G_s)^{-1}K_1 & K_2(1-G_sK_2)^{-1}\tilde{M}_s^{-1} \end{array}\right]$$

The reference model T_0 is chosen as first order of the form

$$T_0 = \frac{G(0)}{0.5s+1} \tag{11}$$

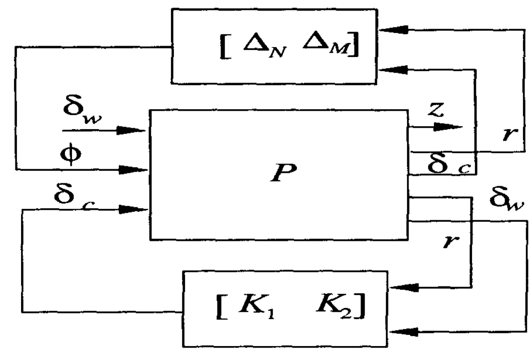

Figure 5: Two-degree-of-freedom associated generalized plant

This model will ensure the same steady state value for the controlled and the conventional car. The settling time is about $1.2(sec)$. The reference model is chosen as a first order in order to avoid any overshot on vehicle responses.

The element M_{11} concerns model matching while element M_{22} constitutes the standard robust stability criterium of coprime factor loop shaping procedure [7]. So the minimization of the norm of M may ensure both good model reference matching and robust stability. A weighting factor can be introduced in order to favor the first or the second requirement [6].

The generalized plant P that corresponds to M is shown on figure 5. It can be easily seen that $M = lft(P,K)$, so standard H_∞ optimization may be used in order to compute the controller K [5].

$$\gamma = \min_{K \text{ stabilising}} \|lft(P,K)\|_\infty \tag{12}$$

H_∞ optimization [5] achieves to $\gamma = 1.94$ and

$$\|M_{11}\|_\infty = 0.50 \tag{13}$$
$$\|M_{22}\|_\infty = 1.12 \tag{14}$$

3.2 Controller implementation

The controller is finally implemented as shown on figure 6. The PI filter is included in the feedback controller K_2. Disturbances are thus filtered by the integral action. However, in order to obtain a similar steady state behavior for the conventional and the controlled car, a speed scheduled gain $\alpha(v)$ has to be added to the prefilter part K_1.

$$\alpha(v) = \frac{G(0,v)}{K_1(0)} \tag{15}$$

Finally the controller implementation needs three type of measurement : measurement of the yaw rate by a gyro, the speed by an odometer and the steering wheel angle by an encoder.

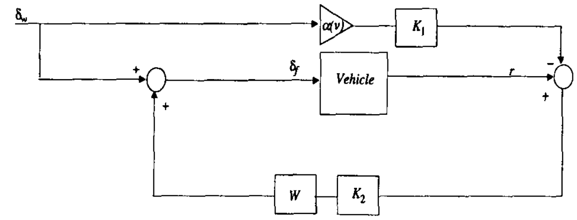

Figure 6: Active steering controller implementation

4 Simulation tests

In all the simulations, solid lines correspond to the controlled car responses and dashed ones to the conventional car responses.

4.1 Disturbance rejection

Firstly a step disturbance wind force of 5000[N] is applied to the vehicle at nominal speed and full road adhesion. It is assumed that the driver doesn't react to this disturbance ($\delta_w = 0$). One can note from figure 7 that the yaw rate goes to zero in steady state, i.e. the yaw angle remains constant in steady state. In comparison, the conventional car presents a non zero steady state yaw rate. This means that the yaw angle presents a ramp increasing without driver reaction. On other hand, the maximum value of yaw rate during the transient phase is smaller than the one of the conventional car and the disturbance is rejected within driver reaction time.

Responses for $v = 30[m.s^{-1}]$ and $\mu = 0.5$ are given in figure 8. The controller exhibits good stability and performance robustness, in fact responses are still well damped.

4.2 Lane change maneuver

The handling improvement is now investigated in case of driver step steering angle which corresponds to lane change maneuver (figure 9-c, dashed line). Figure 9 shows results obtained at nominal speed with common road adhesion equal to 1. Figure 10 shows results obtained for $v = 30\ [m.s^{-1}]$ and $\mu = 0.5$. Due to the speed scheduling of the gain parameter $\alpha(v)$, we ensure that the controlled vehicle and the conventional one present the same steady state behavior. The robust model matching of the previous section makes the controlled vehicle to robustly follow the specified first order reference model T_0. Responses are degraded for high speed and low road adhesion values, however, they practically not present overshot. Figures show also reduction of side slip angle during the transient phase (figure 9-b and 10-b). One can notice that the control effort is limited (figure 9-c).

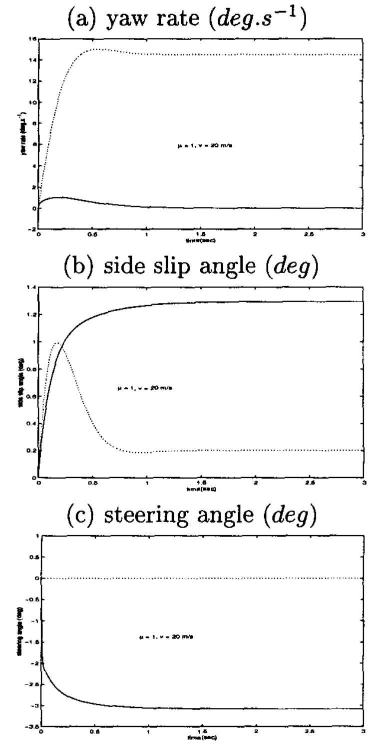

Figure 7: Wind force rejection for nominal system

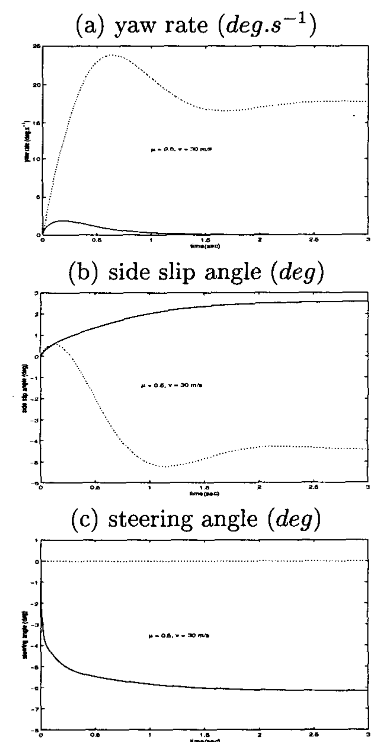

Figure 8: Wind force step input rejection for $v = 30m.s^{-1}$ and $\mu = 0.5$

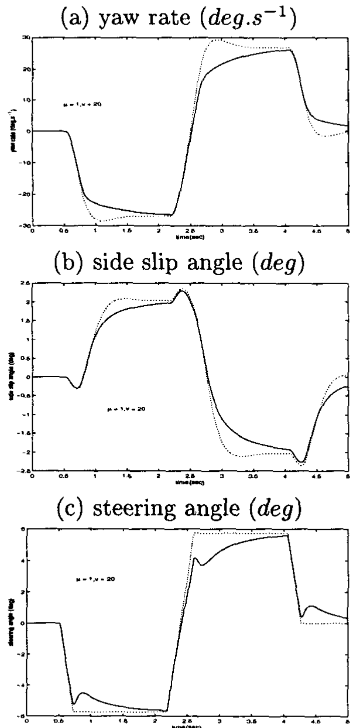

Figure 9: Lane change maneuver, nominal system

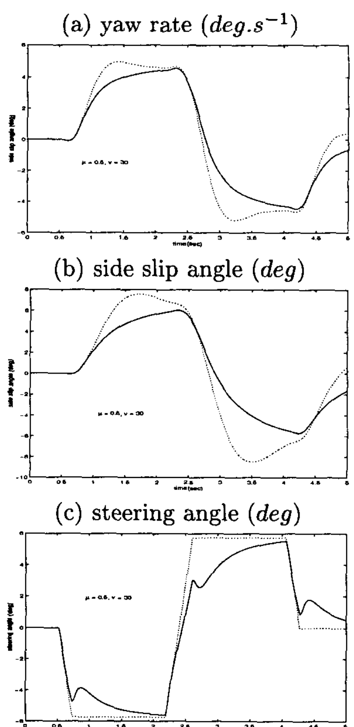

Figure 10: Lane change maneuver for $v = 30m.s^{-1}$ and $\mu = 0.5$

5 Conclusion

In this paper, the two degrees of freedom H_∞ optimization applied to active steering for vehicle handling improvement has been presented. The synthesis methodology simply allows direct specification of time domain objectives. The obtained controller is tested on several typical maneuvers. The controller exhibits good performances and robustness properties face to parameters variations.

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