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Control of vertical flight of the micro aerial vehicle with flapping wings

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Abstract—**Several systems (biomimetic, electric, mechanical...) use means of locomotion based on the iterative movements of the actuators. That results in periodic inputs for the dynamic of these systems. The control of this class of systems by direct application of the existing methods of control does not give periodic inputs inevitably and the consideration of the periodic inputs more complexes the mathematical models of these systems and makes them dependent on time. Our study relates to a particular system which belongs to the class of systems mentioned above which is a Micro Aerial Vehicle with flapping wings. We then propose a control method of this strongly nonlinear system with periodic inputs, based on three principal ideas: the parameterization of the inputs, their taking into account in an implicit way in the model and the application of averaging theory. The averaged model thus released will be used for the calculation of the control by using the method of predictive control, which will be then applied to the original model.**

I. Introduction

THE field of Micro Aerial Vehicle with flapping wings (MAV) represent these last years an interesting field of investigation, due to their interests in both military and civil domains: such small and autonomous devices could be used for inspecting difficult access structures, monitoring of forest fires, or more generally for interventions in narrow and hazardous environments, where it would be dangerous to send a human agent. Concerning the military domain, MAVs prove their interest in being able to be both fully autonomous and dirigible by a single infantryman, with foreseen applications such as rescuing or reconnaissance.

The MAV mainly studied up to now were based on fixed or rotary wings. A model with flapping wings, reproducing the flight of the insects or the hummingbird, was proposed in [1][5][8] in a project carried out to ONERA(Office National d'étude et de recherche aérospatiale) and was validated according to results obtained by the model "Robofly" at the university of Berkeley [2]. The advantages are on one hand a greater maneuverability, particularly at low speeds or even for hovering, and on the other hand a more discrete acoustical spectrum (in comparison with rotary wings).

The model suggested in [1][5][8] is strongly nonlinear and presents periodic inputs. The problem control engineers are confronted with currently is the control of dynamical systems with periodic inputs i.e. to conceive control devices which deliver periodic controls and make it possible to stabilize the machine. In these problems, several works were realized, for example, a technique based on the averaging methods used in electrical engineering, and the application of the backstepping [8] gave extremely encouraging results. Another technique for the control of another model of the MAV with flapping wings proposed in [2] which is based on the classical averaging theory [6]. Always in the class of system with periodic inputs, several studies were carried out on particular systems, we find then in [9][11][12] the control of robots inspired by fish and snake. The technique used is based on the search of an averaged model by using the generalized averaging theory [9][10][13][14].

In what follows, we will work out a strategy of control for the model of the MAV with flapping wings elaborate in [1][5][8]. This model depends on time because it appears in the parameterized inputs. Then by application of the classical averaging theory [6][3][7], an averaged model independent of time is calculated. Basing on this last we are elaborate control laws by application of the predictive control [4]. The laws thus calculated makes it possible to stabilize the original model.

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II. General presentation of the MAV

In this section, we will present the model of the vertical flight of the MAV with flapping wings [1][5][8]. It is put in the following form:

$$\begin{cases} \dot{z} = w \\ \dot{w} = F(w,U,t) = \dfrac{\rho S}{m} V_a^2 (\zeta_{z_s} + \zeta_{z_i}) + g \end{cases} \qquad (1)$$

ρ is the density of the air, S is the surface wing and g is the gravity. State z represents the position of the machine according to the vertical axis, w represents speed according to this same axis. The mass of the machine is defined by m, V_a represents the aerodynamic speed which is equal to

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$V_a = \sqrt{V_{ax}^2 + V_{az}^2}$ and which is a function of aerodynamic speeds according to axis x and z : $V_{ax} = w\cos(\nu) + \dot{\lambda}\sin(\nu)y_F$ according to (ox) and $V_{az} = w\sin(\nu) - \dot{\lambda}\cos(\nu)y_F$ according to (oz).

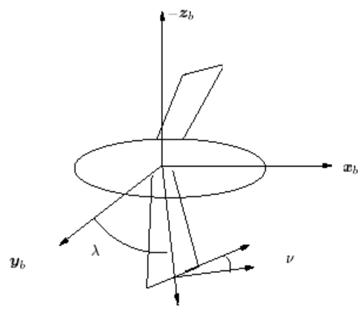


Fig.1. flapping and rotation angles of the wings

The component ζ_{z_s} is related to stationary aerodynamics, and the component ζ_{z_i} is related to non stationary aerodynamics, these two components are defined by:

$$\zeta_{zs} = \cos(\nu)(k_{Ct1}\cos(\alpha) + k_{C_3}\cos(3\alpha)) + \sin(\nu)(k_{Cn1}\sin(\alpha) + k_{C_3}\sin(3\alpha)) \quad (2)$$

$$\zeta_{zi} = \sin(\nu)\left(2\pi\dot{\nu}(1/V_a)c\hat{x}_r + (\pi/2)\ddot{\lambda}(1/V_a^2)cy_F\right) \quad (3)$$

ζ_{z_s} and ζ_{z_i} are expressed according to constants: $k_{Ct1} = 0.27, k_{Cn1} = 3.57, k_{C_3} = 0.1$ [5], angles : $\lambda(t)$ and $\nu(t)$, c is the chord of the profile. $\hat{x}_r$ is defined by: $\hat{x}_r = (3/4) - (l_x / c)$ where l_x is the length between the leading edge and the rotation axis. y_F is the adsciss of the aerodynamic center. And α is the angle of attack which is defined by:

$$\alpha = 2\arctan\left(-V_{az}/(-V_{ax} + \sqrt{V_{ax}^2 + V_{az}^2}\right) \quad (4)$$

The angle of attack for a wing is the angle which forms the chord of its profile (the average line) with the direction of displacement; it is the angle under which the wind comes to strike the profile of the wing. The vector of control is represented by: $U = [\lambda \quad \nu]^T$ where $\lambda(t)$ define variations of the angle of beat of the wing, and $\nu(t)$ define variations of the rotation angle of the latter (fig.1).

III. AVERAGED MODEL

The system that we will control is a strongly nonlinear system with periodic inputs. The method that we will follow is illustrated on (fig.2). Initially, we fix the shape of the periodic inputs that the wings must follow, then we define the new parameters of control. After this stage, we must take these inputs implicitly in the model, which will complexify the model and reveals time in the model. To cross this problem of instationnarity we used the averaging theory which makes it possible to have an equivalent system independent of time which reproduces the same behavior as the original system. After validation of the averaged model by simulations in open loop and definition of the conditions under which the model is validated, we will carry out the application of a method of control which will stabilizes the averaged model and stabilizes the original model.

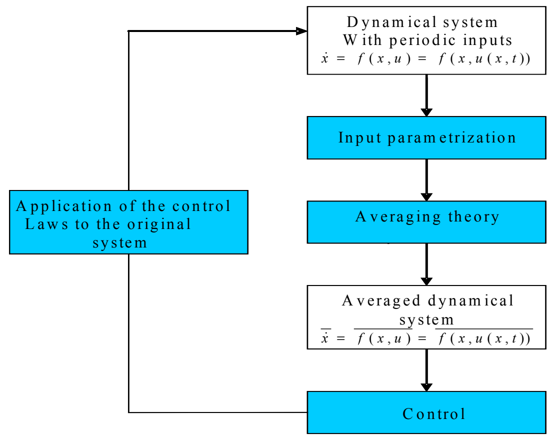


Fig.2. Methodology of control

A. Parameterization of the inputs

For the calculation of the averaged model, we will take a parameterization of the inputs in the form of periodic signals. Studies on the flight of insects show that the variations of the angles of flapping of the wings can be comparable with a triangular signal, and rotations by a square signal [1][2]. After the choice of the form of the periodic inputs, we will define the new parameters of control (amplitudes, frequencies, phase). the actuators used to trail the wings are piezoelectric, which wants to say that the frequency of beat and rotation is fixed (according to studies, the frequency of beat of the wings of the insects turns around 40 Hz), therefore the actual parameters which we will choose for control will be the amplitudes and the dephasing of the two signals. The signal of beat is parameterized in the form:

$$\lambda(t) = \begin{cases} \lambda_p t & 0 \le t < t_{b1} \\ -\lambda_p(t - T/2) & t_{b1} \le t < t_{b2} \\ \lambda_p(t - T) & t_{b2} \le t < T \end{cases} \quad (5)$$

The amplitude of this signal is given by: $\lambda_m = (\lambda_p T)/4$ (λ_p represent the slope). For the rotation of the wings, we will choose a square signal of amplitude ν_m :

$$\nu(t) = \begin{cases} -\nu_m & 0 \le t < t_{r1} \\ +\nu_m & t_{r1} \le t < t_{r2} \\ -\nu_m & t_{r2} \le t < T \end{cases} \quad (6)$$

(Fig.3) illustrates the two signals, their amplitudes and their dephasing. With this parameterization, the new parameters of control are the amplitudes λ_m , ν_m , and the dephasing ϕ which is defined by:

$$\begin{cases} (t_{b1} - t_{r1})/T = \phi/(2\pi) \\ t_{r2} = t_{r1} + T/2 \end{cases} \quad (7)$$

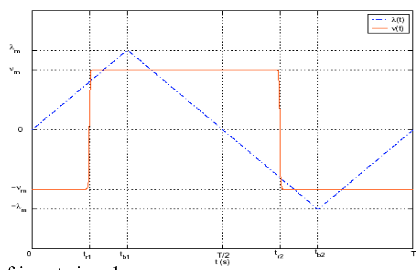


Fig.3. form of input signals

Now, we replace the formulas of these signals in the simplified model of the vertical flight (i.e. taken into account of the inputs in an implicit way), which makes dependent on time and more complex. The averaging theory [2][3] allows to make the system independent on time and in a more handy form for the calculation of the control laws.

B. *Averaging theory*

The essential goals of the averaging theory are to find an approximation independent of time (averaged) of the field of vector f of the differential equation $\dot{x} = f(x,u,t)$ depending on time, and to determine which are the properties which remain unchanged under the effect of the averaging. The most common property is the stability, which is important for the theoretical study of the control.

Let $\dot{x} = f(x,u)$ be a dynamic system with periodic inputs. The input u is necessarily dependent on time considering its periodicity. Then, after parameterization of the periodic shape (triangular, square, sinusoidal...) of the input u and the definition of the new parameters of control, we will have: $u = g(v,t)$, where g is periodic. Thus we reveal in an implicit way time in the model i.e.: $\dot{x} = f(x, g(v,t)) = f(x,v,t)$. To cross the problem of dependence on time of the model, we will apply the averaging theory. From the original system we will have another system independent on time which reproduces the same behavior as the original system:

$$\dot{\bar{x}} = \overline{f(x, g(v,t))}\text{, where: } \overline{f(x,v,t)} = \frac{1}{T}\int_0^T f(x,v,t,)dt$$

And starting from the averaged model, we will calculate the control laws by any method of control and we apply it to the original model. In [3], a control law which stabilizes the averaged model stabilizes also the original model at least on a domain around of operating point.

C. *Averaged model of the MAV with flapping wings*

In this section we will calculate the averaged model if $\phi \geq 0$. This model is represented in the form:

$$\begin{cases} \dot{\bar{z}} = \bar{w} \\ \dot{\bar{w}} = \overline{F(w,U,t)} \end{cases} \tag{8}$$

Where : $\overline{F(w,U,t)} = \frac{1}{T}\int_0^T F(w,U,t)dt = \frac{\rho S}{Tm}\int_0^T V_a^2(\zeta_{zs} + \zeta_{zi})dt + g$

The overlining of variables indicates their averages over one period. It was seen that, since the frequency of the inputs is large, then the dynamic of the system is slow compared to inputs dynamic, which leads us to make the first simplification concerning the state of the system: It is considered then, that the state of the system is fixed over one period and equal to his average (hypothesis 1) (this simplification makes it possible to reduce calculation of the integral over one period, since we do not have the mathematical expression of *w(t)*). For the aerodynamic speed we have:

$$V_a = \sqrt{V_{ax}^2 + V_{az}^2} \tag{9}$$

Where $V_{ax} = w\cos(\nu) + \dot{\lambda}\sin(\nu)y_F$ and $V_{az} = w\sin(\nu) - \dot{\lambda}\cos(\nu)y_F$

Over one period, $\dot{\lambda}(t) = \pm\lambda_p$ thus : $V_a = \sqrt{w^2 + \lambda_p^2 y_F^2}$

What brings us to a calculation much simpler integral :

$$\overline{F(w,U,t)} = \frac{\rho S}{Tm}V_a^2\int_0^T(\zeta_{zs} + \zeta_{zi})dt + g \tag{10}$$

Calculation of $\frac{1}{T}\int_0^T \zeta_{zs}dt$ (component related to stationary aerodynamics): By using trigonometric transformations we will have:

$$\zeta_{zs} = \cos(\nu)\left(k_{Ct1}\cos(\alpha) + k_{C_3}\cos(3\alpha)\right) + \sin(\nu)\left(k_{Cn1}\sin(\alpha) + k_{C_3}\sin(3\alpha)\right)$$

$$\Leftrightarrow \zeta_{zs} = \frac{k_{Ct1} + k_{Cn1}}{2}\cos(\nu - \alpha) + \frac{k_{Ct1} - k_{Cn1}}{2}\cos(\nu + \alpha) + k_{C_3}\cos(\nu - 3\alpha)$$

For reason of simplification of calculations we will identify the quantities $\nu \pm \alpha$ and $\nu - 3\alpha$ (hypothesis 2). After several simulations in open loop, we noted that, for periodic inputs (triangular and square) like defined in the beginning of the paper (Fig.2), the quantities $\nu \pm \alpha$ and $\nu - 3\alpha$ are defined in the following way (in the case $\phi \geq 0$):

$$\nu - \alpha = \begin{cases} -\frac{\pi}{2} & 0 \leq t < t_{b1} \\ +\frac{\pi}{2} & t_{b1} \leq t < t_{b2} \\ -\frac{\pi}{2} & t_{b2} \leq t < T \end{cases}$$

$$\nu + \alpha = \begin{cases} -c_\nu V_m + c_y & 0 \leq t < t_{r1} \\ +c_\nu V_m + c_z & t_{r1} \leq t < t_{b1} \\ +c_\nu V_m - c_y & t_{b1} \leq t < t_{r2} \\ -c_\nu V_m - c_z & t_{r2} \leq t < t_{b2} \\ -c_\nu V_m + c_y & t_{b2} \leq t < T \end{cases}$$

$$\nu - 3\alpha = \begin{cases} +c_\nu V_m - c_a & 0 \leq t < t_{r1} \\ -c_\nu V_m - c_b & t_{r1} \leq t < t_{b1} \\ -c_\nu V_m + c_a & t_{b1} \leq t < t_{r2} \\ +c_\nu V_m + c_b & t_{r2} \leq t < t_{b2} \\ c_\nu V_m - c_a & t_{b2} \leq t < T \end{cases}$$

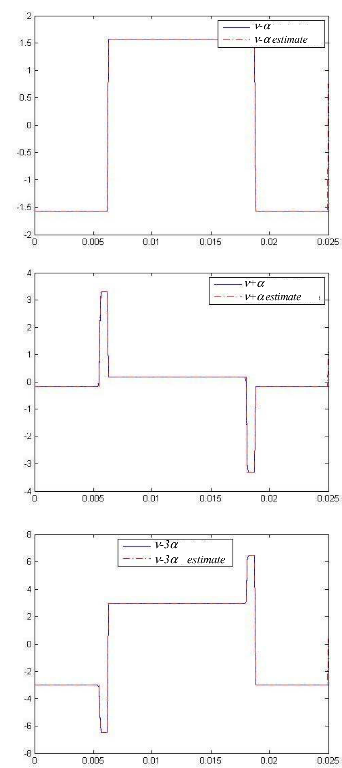


Fig.4 . Identification of the quantities $\nu \pm \alpha$ and $\nu - 3\alpha$

Where constants c_v, c_y, c_z, c_a, c_b are equal to : $c_v = 2$, $c_y = 1.5705$, $c_z = 1.5711$, $c_a = 4.7179$ and $c_b = 4.7128$. These constants are identified by several simulations of the real quantities $\nu \pm \alpha$ and $\nu - 3\alpha$ and their estimates. By using these identifications we calculated the averaged model of the component related to stationary aerodynamics:

$$\frac{1}{T}\int_0^T \zeta_{zs} dt = \frac{k_{Ct1} - k_{Cn1}}{2}[\cos(c_v \nu_m - c_y)(1-\frac{\phi}{\pi}) + \cos(c_v \nu_m - c_z)(\frac{\phi}{\pi})]$$

$$+k_{C_3}[\cos(c_v \nu_m - c_a)(1-\frac{\phi}{\pi}) + \cos(c_v \nu_m + c_b)(\frac{\phi}{\pi})] \qquad (11)$$

Calculation of $\frac{1}{T}\int_0^T \zeta_{zi} dt$ (component related to unstationary aerodynamics): we will suppose it that it is a signal of the form:

$$\begin{cases} -\dfrac{A}{\varepsilon} & t_{r1} - \varepsilon \le t < t_{r1} \\ +\dfrac{A}{\varepsilon} & t_{r2} - \varepsilon \le t < t_{r2} \\ 0 & \textit{ailleurs} \end{cases}$$

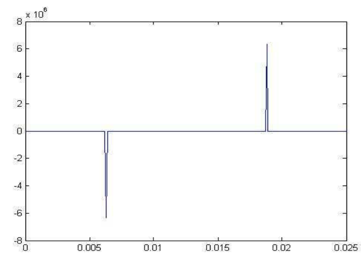


Fig.5. approximation of second derivative of triangular signal

with ε a very small parameter, and A a parameter according to the amplitude of the triangular signal. This signal has the same form with the signal $\ddot{\lambda}(t)$. To identify A, we carried out simulations, which make it possible to calculate the second derivative of $\lambda(t)$ with amplitude λ_m and the signal that we define in the top, then by comparison, we can find that $A = C\lambda_p$ (where C is a constant). The component related to non stationary aerodynamics is given by:

$$\zeta_{zi} = \sin(\nu)\left(2\pi\dot{\nu}(1/V_a)c\hat{x}_r + (\pi/2)\ddot{\lambda}(1/V_a^2)cy_F\right) \qquad (12)$$

The term $\sin(\nu)(2\pi\dot{\nu}(1/V_a)c\hat{x}_r) \approx 0$, since the derivative of $\nu(t)$ is the sum of two impulses which we multiply by the signal $\sin(\nu(t))$ which is square, the moments of commutation correspond to the moments of the two impulses, which enables us to say that this term $\sin(\nu)(2\pi\dot{\nu}(1/V_a)c\hat{x}_r)$ is negligible. On the other hand the second term $\sin(\nu)[(\pi/2)\ddot{\lambda}(1/V_a^2)cy_F]$ is not negligible since the moments of commutations of $\sin(\nu(t))$ are different from the moments of the impulses from $\ddot{\lambda}(t)$, in conditions which dephasing is different from 0 $(\phi > 0)$.

$$\frac{1}{T}\int_0^T \zeta_{zi} dt = \frac{\pi c y_F}{2TV_a^2}(-\frac{A}{\varepsilon}\sin(\nu_m)(t_{b1} + \varepsilon - t_{b1} + \varepsilon)$$

$$-\frac{A}{\varepsilon}\sin(\nu_m)(t_{b2} + \varepsilon - t_{b2} + \varepsilon)) = -\frac{2\pi c y_F}{TV_a^2}(C\lambda_p \sin(\nu_m)) \qquad (13)$$

(where $A = C\lambda_p$)

Then the averaged model of the part related to non stationary aerodynamics is given by:

$$\frac{1}{T}\int_0^T \zeta_{zi} dt = -\frac{2\pi c y_F}{TV_a^2} C\lambda_p \sin(\nu_m) \qquad (14)$$

Then the averaged model can be put in the form:

$$\begin{cases} \dot{\bar{z}} = \bar{w} \\ \dot{\bar{w}} = \dfrac{\rho S}{m}(\bar{w}^2 + \lambda_p^2 y_F^2)[(C1(\cos(c_v \nu_m - c_y)(1-\dfrac{\phi}{\pi}) \end{cases}$$

$$+\cos(c_v \nu_m + c_z)\frac{\phi}{\pi}) + C2(\cos(c_v \nu_m - c_a)(1-\frac{\phi}{\pi})$$

$$+\cos(c_v \nu_m + c_b)\frac{\phi}{\pi}) - \frac{2\pi c y_F C}{TV_a^2}\lambda_p \sin(\nu_m)] + g$$

where : $C1 = (k_{Ct1} - k_{Cn1})/2$, $C2 = k_{C_3}$ and $\lambda_p = (4\lambda_m)/T$

D. *Validation of the averaged model*

We will carry out simulations in open loop of both original and averaged models with the same inputs of control. The following figures show the results:

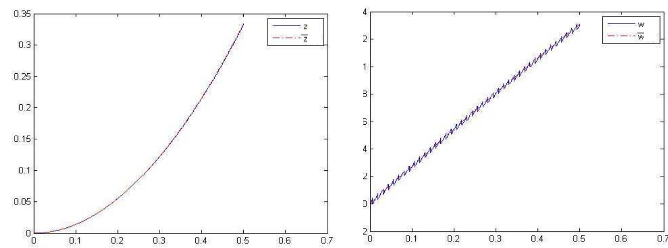


Fig.6. Validation of the averaged model $\lambda_m = 40 / \nu_m = 60 / \phi = 10$ (degrees)

From simulations, we can validate the averaged model. Therefore, we passed from a model depend on time $\dot{x} = f(x,u,t)$ to a model independent of the time which follows the original model perfectly. The only conditions to satisfy so that the model is validated are:

$$0 < \phi < 40^\circ \text{ et } \lambda_m > 0$$

These conditions do not affect our system, considering that the intervals of variations of the parameters λ_m, ν_m, and ϕ do not contain the values quoted above except the value of $\phi = 0$.

We pose:

$$a_{z1} = \frac{\rho S}{m}, a_{z2} = -\frac{8\pi c y_F \rho S C}{T^2 m}, U_\lambda = \lambda_m, U_\nu = \nu_m, U_\phi = \phi/\pi,$$

$$C1 = \frac{k_{Ct1} - k_{Cn1}}{2}, \; C2 = k_{C_3}, \; C3 = \frac{16 y_F^2}{T^2}$$

Then the system will become :

$$\begin{cases} \dot{\bar{z}} = \bar{w} \\ \dot{\bar{w}} = a_{z1}(\bar{w}^2 + C3U_\lambda^2)[(C1(\cos(c_v U_\nu - c_y)(1-U_\phi) \end{cases}$$

$$+\cos(c_v U_\nu + c_z)U_\phi) + C2(\cos(c_v U_\nu - c_a)(1-U_\phi)$$

$$+\cos(c_v U_\nu + c_b)U_\phi)] + a_{z2}U_\lambda \sin(U_\nu) + g$$

The released form enables us, now, to calculate the control laws.

IV. Control of averaged system

The method which we will use for the calculation of the control laws is used already in [4] for the control of the vehicle. the system to be controlled contains two parts in cascade (Fig.7), the control which we will use is based on the minimization of a criterion [4] that we will define starting from the error predicted in one moment *(t+τ)* and controls of the system. The control device will be composed by two controllers: one which minimizes a criterion defines by the error of position *(z-z_c)* and which makes it possible to deliver a control w_c which will be in its turn the instruction for the second subsystem. The controls u will be calculated in this case by minimizing a criterion, but this time defined by the error speed *(w-w_c)*.

A. Calculation of the controls

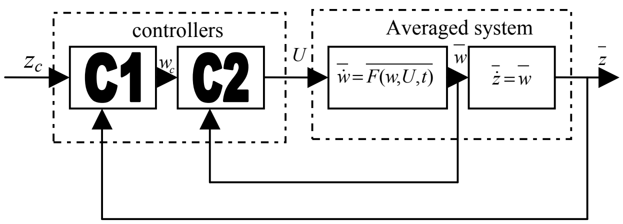


Fig.7. control of averaged system in closed loop

1) Controller C1

We will now define the criterion J_1 to be minimized:

$$J_1 = e_z(t+\tau_z)^T Q e_z(t+\tau_z) + w^T R w$$
$$e_z(t) = z(t) - z_c(t)$$

τ_z is a parameter of prediction. Minimization also relates to the speed w which is regarded as a virtual control for the second subsystem. Since the value $e_z(t+\tau_z)$ and w are scalars then the criterion will be written in the form:

$$J_1 = Qe_z(t+\tau_z)^2 + Rw^2$$

While developing $e_z(t+\tau)$ in infinite series, we will have:

$$e_z(t+\tau_z) = e_z(t) + \tau_z \dot{e}_z(t) + \frac{\tau_z^2}{2}\ddot{e}_z(t) + ...$$

In our application we will stop at order 1, which leads us to:

$$J_1 = Q(e_z(t) + \tau\dot{e}_z(t))^2 + Rw^2$$
$$J_1 = Q((z-z_c) + \tau(\dot{z}-\dot{z}_c))^2 + Rw^2$$

We have $\dot{z} = w$ according to the model :

$$J_1 = Q((z-z_c) + \tau(w-\dot{z}_c))^2 + Rw^2$$

To minimize this criterion compared to w, it is derived and calculate the value of w which cancels the derivative and consequently minimizes the J_1 criterion:

$$\frac{\partial J_1}{\partial w} = Q(2\tau_z^2(w-\dot{z}_c) + 2\tau(z-z_c)) + 2Rw$$
$$\frac{\partial J_1}{\partial w} = (2\tau_z^2 Q + 2R)w - Q(2\tau_z^2\dot{z}_c - 2\tau_z(z-z_c))$$
$$\frac{\partial J_1}{\partial w} = 0 \Leftrightarrow (2\tau_z^2 Q + 2R)w - Q(2\tau_z^2\dot{z}_c - 2\tau_z(z-z_c)) = 0$$

The solution of this equation gives us the control law w_c, which makes it possible to bring back state z to the reference

$$w_c = Q(2\tau_z^2\dot{z}_c - 2\tau_z(z-z_c))/(2\tau_z^2 Q + 2R) \qquad (15)$$

The control w_c represents a reference for the controller 2, who will calculate the control laws U to bring back the speed of the system to follow the reference w_c.

2) Controller C2

In the same way we define a criterion for the second subsystem which describes the evolution speed, therefore the state indicates speed w, and the controls are the amplitudes of the signals (triangular and square) and the dephasing between these two signals. The criterion to be minimized is form:

$$\begin{aligned} &J_2 = e_w(t+\tau_w)^T P e_w(t+\tau_w) + U^T B U \\ &e_w(t) = w(t) - w_c(t) \\ &U = \begin{bmatrix} U_\lambda & U_\nu & U_\phi \end{bmatrix} \end{aligned} \qquad (16)$$

τ_w is a parameter of prediction. P is a scalar, and B a diagonal matrix of dimension 3×3 of the form:

$$B = \begin{bmatrix} B_1 & 0 & 0 \\ 0 & B_2 & 0 \\ 0 & 0 & B_3 \end{bmatrix}$$

While developing $e_w(t+\tau_w)$ in infinite series, and replacing it in the equation (16), we will have:

$$\begin{aligned} J_2 = {} & P(e_w(t))^2 + \tau_w^2(\dot{w}-\dot{w}_c)^2 + 2\tau_w e_w(t)(\dot{w}-\dot{w}_c)) \\ & + B_1U_\lambda^2 + B_2U_\nu^2 + B_3U_\phi^2 \end{aligned}$$

We have also :

$$\begin{aligned} \bar{w} = {} & a_{z1}(\bar{w}^2 + C_3U_\lambda)[(C_1(\cos(c_vU_\nu - c_y)(1-U_\phi) \\ & + \cos(c_vU_\nu + c_z)U_\phi) + C_2(\cos(c_vU_\nu - c_a)(1-U_\phi) \\ & + \cos(c_vU_\nu + c_b)U_\phi)] + a_{z2}U_\lambda\sin(U_\nu) + g \end{aligned} \qquad (17)$$

After taking into account of the conditions $U_\phi > 0$ and $U_\lambda > 0$, and the replacement of equation (17) in J_2, we used the *Matlab* constrained minimization algorithm "*lsqnonlin*". So, the control laws that we applied to the system are the result of the minimization of the criterion J_2 under constraints.

B. Results of simulation

In this section, we will have some results of simulation of the averaged model of the MAV with flapping wings. Because of physical constraints on the wings, we will impose an interval of variation for each control [5] while taking into account the conditions of validity of the averaged model, therefore that wants to say that controller 2 consists of the minimization of the J_2 criterion, under the constraints on the controls. The control laws λ_m, ν_m and ϕ, belong to the following intervals (in degrees):

$$\lambda_m \in [40,80],\ \nu_m \in [40,80] \text{ et } \phi \in [05,30]$$

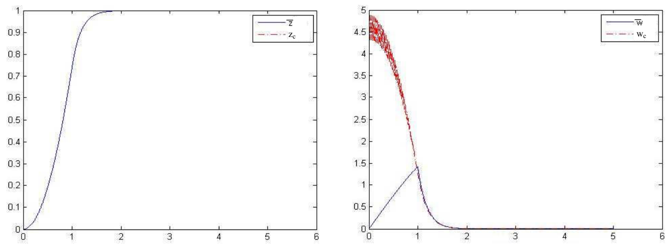


Fig.8 : Position and speed of the averaged model

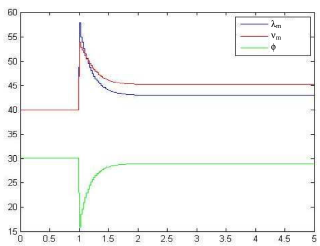


Fig.9. form of the controls of the averaged model (amplitudes and dephasing)

V. CONTROL OF THE ORIGINAL MAV MODEL

Now, after having calculated the control laws of the vertical flight by using the averaged model, we will apply them to the original model. The control laws are calculated and applied at each beginning of period, they are fixe on all the interval *[T,t+T]*. The results of simulation are illustrated on the figure (10). We notice that with the same parameters of adjustment, we had the same behaviors, for the original model and the averaged model (even response time and precision).

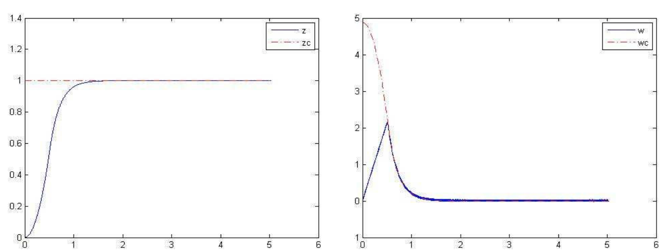
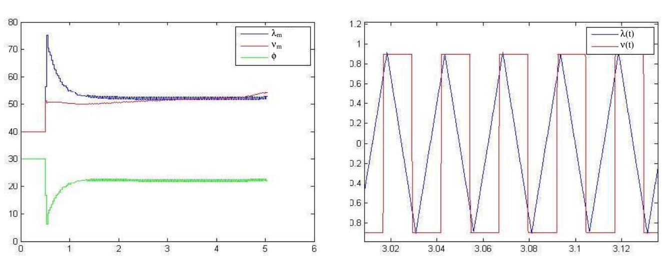


Fig.10 : position and speed of the original system



(a) (b)

Fig.11 : (a) variations of the amplitudes and dephasing of the two signals -(b) real form of the signals

It is also notable that the controls (amplitudes and dephasing of the two signals) (Fig.11.a) are realizable (not chattering) from the technological point of view. The real shapes of the inputs of control are illustrated in (Fig.11.b).

VI. CONCLUSIONS AND PERSPECTIVES

In this article, we applied a step of control to a Micro Aerial Vehicle with flapping wings modeled by Th.Rakotomamonjy [5] (we are interested in the vertical flight of the MAV). The model being strongly nonlinear with periodic inputs. The first stage of the strategy of control is the parameterization of the inputs in the form of periodic signals and definition of new parameters of control (amplitudes and dephasing). Afterwards, we introduced these signals into the mathematical model of the MAV. The introduction of these functions depending on time into the model let appear explicitly the time variable. The second stage being to replace the time-depending model to a model independent of the time which approximates the behavior of the system, and that by applying the classical averaging theory. The calculation of the control laws is made by minimization of two criteria. The found results are very interesting, and the control proved to be robust in the presence of disturbances. Future work will relate to the application of this strategy of control for all the cases of flight of the MAV and the application of others methods of control to compare the results.

VII. REFERENCES

[1] Th. Rakotomamonjy, Th. Le Moing, M. Ouladsine. *Simulation Model of a Flapping-wing Micro Air Vehicle*. EMAV 2004, Braunschweig, Allemagne, juillet 2004.

[2] Luca Schenato, *Analysis and Control of Flapping Flight: from Biological to Robotic Insects*, Ph.D. thesis, university of California at Berkeley, 2003

[3] Domenico Campolo, Luca Schenato, Eugenio Guqlielmelli, Shankar S. Sastry, *A Lyapunov-based approach for the control of biomimetic robotic systems with periodic forcing inputs,* IFAC, 2005

[4] H. Shraim, M. Ouladsine, M. El Adel, *A New Nonlinear Control Strategy for a vehicle trajectory tracking in the presence of faults*, in: 44th IEEE Conference on Decision and Control CDC and European Control Conference ECC 2005, Seville, Spain, 12-15 December 2005.

[5] Thomas Rakotomamonjy, *Modélisation et contrôle du vol d'un microdrone à ailes Battantes*, Ph.D. thesis university Paul Cézanne, Aix-Marseille III, January 2006

[6] Jan A Sanders, Ferdinand Verhulst, *Averaging Methods in Nonlinear Dynamical Systems*, Edition Springer, 1985.

[7] Xinyan Deng, Luca Schenato, Shankar S. Sastry, *Flapping Flight for Biomimetic Robotic Insects: Part II-Flight Control Design* à paraître in IEEE Transactions on Robotics.

[8] Th. Rakotomamonjy, Th. Le Moing, M. Ouladsine. *Développement d'un modèle de simulation d'un microdrone à ailes vibrantes*. CIFA 2004, Douz, Tunisia, November 2004.

[9] Patricio Antonio Vela, *Averaging and Control of Nonlinear Systems (With Application to Biomimetic Locomotion)*, Ph.D. thesis California Institute of Technology Pasadena, California, Mai 2003

[10] Patricio Vela, Kristi A. Morgansen, Joel W.Burdick, *Second order averaging methods for oscillatory control of underactuated mechanical systems*, IEEE American Control Conference, 2002.

[11] Vela P.A., Morgansen K.M., and Burdick J.W. Underwater Locomotion From Oscillatory Shape Deformations, *Proceedings IEEE Conference on Decision and Control*, 2002.

[12] Morgansen K.M., Vela P.A., and Burdick J.W. Trajectory Stabilization for a Planar Carangiform Robot Fish, *Proceedings IEEE Conference on Robotics and Automation*, 2002
[13] Vela P.A. and Burdick J.W. Control of Biomimetic Locomotion via Averaging Theory, *to Appear 2003 IEEE Conference on Robotics and Automation*, 2002.
[14] P. Vela and J.W. Burdick, *A General Averaging Theory via Series Expansions* , 2003 American Control Conference