PI Observer Robust Fault Estimation for Motorcycle Lateral Dynamics

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***Abstract*—This paper is devoted to the robust fault detection and estimation for the lateral dynamics of a motorcycle. The later is modelled using an uncertain switched system formalism. A switched proportional integral observer is designed in order to minimize the effect of the disturbance and uncertainties on residual sensitivity. It consists in the design of proportional integral observer which minimizes the well-known H_∞ norm and ensures Input-to-State-Stability (ISS). The problem is formulated as a linear matrix inequalities (LMI) feasibility problem in which a cost function is minimized subject to LMI constraints. Various estimation problems are considered including state estimation, unknown input estimation and non-linear dynamics behaviour estimation which is considered as a fault input. A set of simulation studies are provided in order to establish the validity of the approach which allows sensor less implementation of driving assistance systems for motorcycles.**

I. INTRODUCTION

Passive and active driving assistance systems have been more addressed for four-wheel vehicles than for single track-vehicles. The results is that several functions have become standard in today four-wheeled vehicles. They lead to a reduction of road accidents and fatalities. However, single track vehicles are more and more attractive. Besides the fun aspect, they are chosen to deal with hard traffic conditions in urbanized areas which also suffer from parking problems. This causes an increase of two-wheeled vehicles traffic and this type of vehicle is facing increased accident and death statistics. Several research projects have been launched during the last years nationally and internationally in order to analyse the process of two-wheeled vehicles accidents and to propose preventive and active safety systems [1].

Driving assistance systems implemented on four-wheeled vehicles make use of information provided by several embedded sensors. In addition, observers are generally implemented in order to estimate unmeasurable variables such as the lateral velocity. Implementing sensors on single-track vehicles is even more complicated due to the reduced available space. In the literature, observer design has been addressed for different issues. In this framework several motorcycle models have been proposed in the literature for dynamics analysis [4]. They have been also used for observer synthesis and control design [5],[8],[17]. When considering the estimation of the lateral dynamics, the longitudinal speed is generally assumed to be constant. However, this assumption is only true is steady state cornering conditions and it leads to poor estimation performance [7], [10], [9].

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On the theoritical part, state estimation has been addressed for several system classes going from linear time-invariant to non-linear ones [14]. In order to face system uncertainties, disturbances and unknown input affecting the system, different types of observers have been developed, among them, robust and unknown input observers. Systems are also affected by actuator and sensor faults. For safety reasons and in order to maintain system functionality, several papers are concerned with detection and diagnosis of process faults [13], [12], including for uncertain [14] and discrete systems. Approaches include fault detection, isolation and estimation. A residual signal is generally computed in order to represent the inconsistency between the actual plant variables and the assumed model, to extract information on possible changes caused by faults. One challenging goal is to generate an insensitive residual against noises and uncertainties and still sensitive to faults [13].

This paper aims to propose an observer design procedure which takes into account the time variation of the longitudinal speed and estimates both the state and the extra variables that enter the model as faults. These extra variables represent unknown inputs or the difference between the assumed linear behaviour and the real non-linear behaviour of the tire forces. It is proposed to use the switching system modelling of the motorcycle Linear Parameter Varying model. It offers the possibility to use Multiple Lyapunov Functions (MLF) for the design of a Proportional Integral (PI) robust observer for the estimation of faults while minimizing the effect of the disturbance input on the residual signal. The problem is formulated as a convex optimization problem under Linear Matrix Inequalities (LMI) conditions.

The outline of this paper is as follows. After the introduction, the motorcycle model is presented and some observability criteria are discussed in Section II. The state and the fault estimation problems are formulated therein. In Section III, the design procedure of the H_∞ robust switched PI observer is addressed. It ends with the problem formulation as an optimization problem under LMI constraints. It includes Input-to-State-Stability criteria (ISS). Simulation studies under various lateral dynamics solicitations are conducted in Section IV. Section V provides some concluding remarks and extension in future work.

Notations: The following notation is used in the paper. For any matrix Z^T is the transposed of matrix Z, the star symbol $(\star)$ in a symmetric matrix denotes the transposed block in the symmetric position. The notation $P \succeq (\preceq) 0$ means P is a real symmetric positive (negative) semi-definite matrix. 0 and

I denote zero and identity matrix of appropriate dimensions.

II. MOTORCYCLE MODEL FOR FAULT ESTIMATION

A. Lateral dynamics equations

Motorcycle dynamics are highly non-linear and more complex to obtain, comparing to four-wheel vehicle dynamics. However, decomposition into in-plane (longitudinal) and out-plane dynamics (lateral) is still possible when we are interested in specific dynamics or maneuvers study [3], [11], [4]. This paper is focussed on the lateral dynamics. The state components of the simple linear model includes: the lateral velocity v_y, the yaw rate $\dot{\psi}$, the roll rate $\dot{\phi}$, the handlebar steering angle δ and its derivative $\dot{\delta}$, the front and the rear wheel lateral tire forces F_f and F_r respectively. The lateral forces are included in the state vector in order to take into account of the relaxation phenomena which is important for the dynamics representation. The state vector reads: $x = \left[v_y, \dot{\psi}, \dot{\phi}, \delta, \dot{\delta}, F_f, F_r\right]^T$. The lateral dynamics equations are:

$$\begin{cases} M(\dot{v}_y + v_x\dot{\psi}) + M_f k\ddot{\psi} + (M_f j + M_r h)\ddot{\phi} \\ +M_f e\ddot{\delta} = F_f + F_r \\ M_f k(\dot{v}_y + v_x\dot{\psi}) + a_2\ddot{\phi} + a_3\ddot{\psi} + a_1\ddot{\delta} - a_4 v_x\dot{\phi} \\ -\frac{i_{fy}}{R_f}\sin\varepsilon v_x\dot{\delta} = \sum M_z \\ (M_f j + M_r h)\dot{v}_y + b_2\ddot{\phi} + a_2\ddot{\psi} + b_1\ddot{\delta} + b_5 v_x\dot{\psi} \\ +\frac{i_{fy}}{R_f}\cos\varepsilon v_x\dot{\delta} = \sum M_x \\ M_f e\dot{v}_y + b_1\ddot{\phi} + a_1\ddot{\psi} + c_1\ddot{\delta} + c_3 v_x\dot{\psi} \\ -\frac{i_{fy}}{R_f}\cos\varepsilon v_x\dot{\phi} + K\dot{\delta} = \sum M_s \end{cases} \quad (1)$$

where

$$\begin{cases} \sum M_z = l_f F_f - l_r F_r \\ \sum M_x = b_4 \sin\phi - b_3 \sin\delta \\ \sum M_s = -b_3\sin\phi - c_2\sin\delta - \eta F_f + \tau \end{cases} \quad (2)$$

The model parameters numerical values are provided in the appendix.

The tire forces equations are governed by the equations:

$$\frac{\sigma_f}{v_x}\dot{F}_f + F_f = F_{sf} \quad (3)$$

$$\frac{\sigma_r}{v_x}\dot{F}_r + F_r = F_{sr} \quad (4)$$

The index (f) and (r) are used to identify variables related to the front tire and rear tire respectively. The static forces F_{sf} and F_{sr} are governed by the pacejka non-linear model [2]. The front and rear tire lateral forces are function of the front and the rear sideslip angles the tire-road contact location which are denoted α_f and α_r respectively and the front and rear camber angles denoted $\gamma_f = \phi + \delta\sin\varepsilon$ and $\gamma_r = \phi$ respectively. Finally σ_f and σ_r are the front and the rear tire relaxation length respectively. In order to catch the tire saturation, the tire forces model is split into linear and non-linear parts:

$$F_{sf} = c_{f1}\alpha_f + c_{f2}\gamma_f + \Gamma_f \quad (5)$$

$$F_{sr} = c_{r1}\alpha_r + c_{r2}\gamma_r + \Gamma_r \quad (6)$$

where Γ_f and Γ_r are the non-linear part of the tire forces. In the normal zone, the lateral forces are linear and these terms are zero. In the critical zone, these terms rise and could not be neglected. In the sliding zone, the terms become almost constant. The coefficients c_{f1}, c_{f2}, c_{r1} and c_{r2} are the lateral and camber tire stifnesses. The lateral front and rear sideslip angles are thus given by

$$\alpha_f = -\frac{v_y + l_f\dot{\psi} - \eta\dot{\delta}}{v_x} + \delta\cos\varepsilon \quad (7)$$

$$\alpha_r = -\frac{v_y - l_r\dot{\psi}}{v_x} \quad (8)$$

where, η is the mechanical trail, ε is the rake angle of steering set, l_f and l_r are the distances from CG to the front and rear tires respectively. Notice that the expressions of these angles are normally non-linear, but the saturation of the forces is reached for low values of these angles, which makes it possible to adopt a linearised form.

B. State-space model

The previous lateral model could be written under the form of a Linear Parameter Varying model where the varying parameter is the forward speed v_x which appears as a parameter in the model equations.

$$\begin{cases} \dot{x}(t) = \bar{A}(v_x)\,x(t) + B(v_x)\,u(t) \\ +E_\phi(v_x)\,\phi(t) + E_\Gamma(v_x)\,\Gamma(t) \\ y(t) = Cx(t) + F_d(v_x)\,d(t) + F_\Gamma(v_x)\,\Gamma(t) \end{cases} \quad (9)$$

where $x(t)$ is the state vector, $u(t) = \tau(t)$ is the control input, $\phi(t)$ is the roll angle input, $\Gamma(t) = [\Gamma_f(t), \Gamma_r(t)]^T$ is the vector of forces non-linearities and $y(t)$ is the measurement vector. All the matrices are assumed to be of compatible dimensions. The roll angle enters the model as an input. In fact including the roll angle in the state space vector renders the model not observable (detectable). In this case an exteroceptive sensor measuring the roll angle is needed.

Single Axle vehicle stability is not ensured for the entire speed range. Stabilisation by rider actions on the handlebars and body motion is required. Nevertheless, the considered model is stable for the speed range $[10, 30]\ m/s$.

The tires lateral stiffness coefficients c_{f1}, c_{f2}, c_{r1}, c_{r2} are subject to variations according to the road conditions. Their values are unknown, but deviation from nominal values are bounded. As these coefficients appear in the matrix $\bar{A}(v_x)$, this matrix is thus considered as uncertain. Its variations are also bounded and can modelled under the form of additive uncertainty:

$$\bar{A}(v_x) = A(v_x) + \Delta A(v_x) \quad (10)$$

where $\Delta A(v_x)$ is the disturbance matrix which verifies $\Delta A(v_x)^T \Delta A(v_x) \leq \theta\ \mathrm{I}$, where θ is a positive bounding constant which is of finite value since the system is stable.

Let us now model the A matrix uncertainty as an additional disturbance input. Denoting $\xi(t) = \Delta A(v_x)\,x(t)$, since the motorcycle is stable and the matrix dynamics uncertainties bounded, $\xi(t)$ is a bounded input. The system of equation (9) could be rewritten as:

$$\begin{cases} \dot{x}(t) = A(v_x)\,x(t) + B(v_x)\,u(t) \\ +E_\phi(v_x)\,\phi(t) + \xi(t) + E_\Gamma(v_x)\,\Gamma(t) \\ y(t) = Cx(t) + F_d(v_x)\,d(t) + F_\Gamma(v_x)\,\Gamma(t) \end{cases} \tag{11}$$

C. Observability and used sensors

The observability of the system is directly linked to the used sensors and the structure of the model. For single track vehicles two main constraints have to be taken into account. Firstly the available place for sensor placement is very limited. Secondly, the sensors should be compatible with admissible costs by the motorcycle industry. For example, the measurement of the handlebars torque and the lateral velocity have to be avoided as the first one requires a physical modification of the motorcycle and the available technology for the second one is incompatible with common motorcycle cost (several thousand euros). The measurement of the roll angle requires an exteroceptive sensor such as a video or a laser sensor. An inertial unit gives access to the yaw and the roll velocities and the lateral acceleration. Steering angle could be obtained from an optical encoder mounted on the motorcycle fork.

The state-space model presented above has a state-vector with 7 components while the driver torque and the motorcycle roll angle are unknown or known model inputs. Two cases have to be distinguished:

- These two inputs are measured. The system is observable using classical proprioceptive sensors if at least two measurements are used. Naturally, the measurement of the lateral velocity and the lateral forces could not be used while the couple $\left(\delta, \dot{\delta}\right)$ is not suitable. Notice that the lateral acceleration is a potential measurement $\gamma_{lat} = \frac{1}{M}\left(F_f + F_r\right)$. The state could be estimated, the remaining problem is the estimation of the non-linear part of the tire forces while minimizing the effect of the model uncertainties.
- The roll angle and the rider torque are not measured. The state estimation should be considered either in the framework of the minimization of the effect of these inputs and the other ones or in the framework of the combination of state and unknown input estimation. Unfortunately, in this case only the first framework is possible with a 7th order model. One has to reduce the model in order to ensure potential for unknown input estimation which are denoted faults in this paper.

D. Model reduction

As explained above, the torque and the roll angle are assumed to be unknown. From the model formulation, the system given in equation (9) presents a total of 5 exogenous signals. According to the requirements one may need to estimate a given input while minimizing the effect of the others. For example, the driver torque estimation may be required while this estimation has to be insensitive to tires non-linearities, roll angle and parameters variations.

Unfortunately, with the previous state-space model, it is not possible to estimate $\tau(t)$, $\phi(t)$ and $f(t)$ using the integral action, as the augmented system is not observable. In order to overcome this problem, the tire relaxation lengths σ_f and σ_r are set to zero. This allows to reduce the state vector to $x = \left[v_y, \dot{\psi}, \dot{\phi}, \delta, \dot{\delta}\right]^T$.

E. From LPV to switched model

The longitudinal speed appears in the model as a measured parameter varying. Generally, a polytopic approach with extremal models corresponding to the minimum and the maximum values of the speed. Takagi-Sugeno models are also used [6], allowing to obtain a continuous transition model from submodels thanks to the premise variables. However, these approaches are known to be conservative due to the large speed variation domain. Instead, in the following, a switched model approach is chosen. In this case, several models are defined for different speed values and the model switches form one subsystem to another according to the speed variations. The model equation (9) is rewritten under the form:

$$\begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma u(t) \\ + \begin{bmatrix} E_{d,\sigma} & I \end{bmatrix} \begin{bmatrix} d(t) \\ \xi(t) \end{bmatrix} + E_{f,\sigma} f(t) \\ y(t) = Cx(t) + F_{d,\sigma} d(t) + F_{f,\sigma} f(t) \end{cases} \tag{12}$$

where $\sigma(t)$ is the switching parameter which is assumed to be known. This parameter is obtained as the index of the forward speed grid regions obtained by the discretization the speed range $[v_{\min}, v_{\max}]$. Given $v_{\min} = v_1 \leq v_2 \leq ... \leq v_N = v_{\max}$, any given speed $v(t) \in [v_\sigma, v_{\sigma+1}]$ and $\sigma \in \{1, 2, ..., N-1\}$.

At this stage, the model (12) could be considered as general. In fact a classical observation problem addresses the estimation of the state vector $x(t)$ and the fault vector $f(t)$ assuming that the input vector $u(t)$ and the output $y(t)$ are available. The estimation has to be robust to the disturbance input $d(t)$ which is assumed to be unknown. The fault vector could include actuators faults, sensor faults or any other signals that should be estimated. In addition one can estimate only one or more variables by putting those to be estimated in $f(t)$ and the others against which the estimation has to be insensitive in $d(t)$. Two specific situations are provided here for better clarification.

- The rider torque is measured $u(t) = \tau(t)$, one wants to estimate $f(t) = \phi(t)$ while minimizing the effect of $\tilde{d}(t) = \begin{bmatrix} d(t) \\ \xi(t) \end{bmatrix}$ with $d(t) = \begin{bmatrix} \Gamma_f(t) \\ \Gamma_r(t) \end{bmatrix}$.
- The rider torque is not measured $u(t) = 0$, one wants to estimate it $f(t) = \tau(t)$ while minimizing the effect of $\tilde{d}(t) = \begin{bmatrix} d(t) \\ \xi(t) \end{bmatrix}$ with $d(t) = \begin{bmatrix} \phi(t) \\ \Gamma_f(t) \\ \Gamma_r(t) \end{bmatrix}$.

Finally, it is not yet decided which signal is included in vector $d(t)$ and which affected to $f(t)$. One only has to consider that the components of these vectors could be τ, ϕ, Γ_f and Γ_r, and $n_d + n_f = 4$.

III. SWITCHED PI OBSERVER

PI observers have been widely used in the litterature [15]. Assuming that the signals $f(t)$ to be estimated are constant, one can use a Proportional Integral Observer (PIO) in order to obtain a relevant estimate. It is wellknown that even if these inputs are not constant, the observer still works well by adjusting the observer bandwith [16]. Another possibility is to use Multiple Integral Proportional Observers which are able to estimate varying unknown inputs [15].

In the following, we consider the design on a switched robust PI observer for system (12) given by the equations [16].

$$\begin{cases} \dot{\hat{x}}(t) = A_\sigma \hat{x}(t) + B_\sigma u(t) + L_{P,\sigma}\left(y(t) - \hat{y}(t)\right) + E_{f,\sigma}\hat{f}(t) \\ \dot{\hat{f}}(t) = L_{I,\sigma}\left(y(t) - \hat{y}(t)\right) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{13}$$

where $\hat{x} \in \mathbb{R}^{n_x}$ is the state estimate, $\hat{y} \in \mathbb{R}^{n_y}$ is the measurement estimate, $\hat{f} \in \mathbb{R}^{n_f}$ is the fault vector input estimate. The observer proportional matrix gain is $L_{P,\sigma} \in \mathbb{R}^{n_x \times n_y}$ and the integral matrix gain is $L_{I,\sigma} \in \mathbb{R}^{n_f \times n_y}$. The Observer gain is $L_\sigma^T = \left[L_{P,\sigma}^T, L_{I,\sigma}^T\right]^T$.

The PI observer will be designed with the aim to reduce to effect of the disturbance input $d(t)$ on the residual which is defined for each mode as:

$$r_\sigma(t) = y(t) - \hat{y}(t) \tag{14}$$

This residue is defined for each sub-system since the estimated output is obtained from different observers.

A. Design procedure

The observer design needs the definition of the state estimation error $e_x = x - \hat{x}$ and the fault estimation error $e_f = f - \hat{f}$. These two errors will be also considered together into the augmented error vector $\tilde{x} = \left[e_x^T, e_f^T\right]^T$.

The error vector dynamics and the residual are given by:

$$\begin{cases} \dot{\tilde{x}} = \left(\tilde{A}_\sigma - L_\sigma \tilde{C}_\sigma\right)\tilde{x} + \left(\tilde{E}_{d,\sigma} - L_\sigma F_{d,\sigma}\right)d \\ -L_\sigma F_{f,\sigma} f + \tilde{I}\xi \\ r_\sigma = \tilde{C}_\sigma \tilde{x} + F_{d,\sigma} d + F_{f,\sigma} f \end{cases} \tag{15}$$

where $d \in \mathbb{R}^{n_d}$ and the matrices are defined by

$$\tilde{A}_\sigma = \begin{bmatrix} A_\sigma & E_{f,\sigma} \\ 0 & 0 \end{bmatrix}, \tilde{B}_\sigma = \begin{bmatrix} B_\sigma \\ 0 \end{bmatrix}, \tilde{E}_{d,\sigma} = \begin{bmatrix} E_{d,\sigma} \\ 0 \end{bmatrix}$$
$$\tilde{C}_\sigma = \begin{bmatrix} C & 0 \end{bmatrix}, \tilde{I} = \begin{bmatrix} I \\ 0 \end{bmatrix}, L_\sigma = \begin{bmatrix} L_{P,\sigma} \\ L_{I,\sigma} \end{bmatrix}$$

The objective of the H_∞ switched fault detection PI observer is to ensure that the sensitivity transfer function from the disturbance input $[d^T \xi^T]^T$ to the residual r_σ verifies

$$\left|\left|T_{r_\sigma \tilde{d}}\right|\right| < \gamma_\sigma \tag{16}$$

where $\tilde{d} = \begin{bmatrix} d^T \xi^T \end{bmatrix}^T$ and γ_σ are positive numbers. One has to notice that γ_σ is seek for each switched subsystem. The achieved values may be different for each mode. The minimization is performed in the case of zero fault.

Let $A_\sigma^* = \tilde{A}_\sigma - L_\sigma \tilde{C}_\sigma$, $E_{d,\sigma}^* = \tilde{E}_{d,\sigma} - L_\sigma F_{d,\sigma}$, $E_{f,\sigma}^* = -L_\sigma F_{f,\sigma}$.

In the following, the input ξ is only assumed to be norm bounded, the equation (15) is used in the development of the sufficient LMI conditions for robust PI observer fault detection.

B. H_∞ robust PI fault estimation observer

According to the matrices defined above, the error dynamics could be written as

$$\dot{\tilde{x}} = A_\sigma^* \tilde{x} + E_{d,\sigma}^* d + E_{f,\sigma}^* f + \tilde{I}\xi \tag{17}$$

Considering the Multiple Lyapunov Function

$$V_\sigma = \tilde{x}^T P_\sigma \tilde{x} \tag{18}$$

The Lyapunov function has to verify the inequality

$$\dot{V}_\sigma + r_\sigma^T r_\sigma - \gamma_\sigma^2 \tilde{d}^T \tilde{d} < 0 \tag{19}$$

In order that equation (16) is verified. This condition is transformed into a Linear Matrix Inequality involved in an optimization problem.

In fact, if there exist symmetric definite matrices P_σ, U_σ, scalars β_σ, scalars $\bar{\alpha}_\sigma$, and positive scalars γ_σ, solving the optimization problem for each mode $(\sigma = 1, 2, ..., N)$ of the switched system [16]

$$\min_{P_\sigma, U_\sigma, \bar{\gamma}_\sigma, \bar{\alpha}_\sigma} \beta_\sigma \bar{\alpha}_\sigma + (1 - \beta_\sigma)\bar{\gamma}_\sigma$$

under

$$\begin{pmatrix} \Omega_{d,\sigma} & \Lambda_{d,\sigma} & P_\sigma \tilde{I} \\ * & F_{d,\sigma}^T F_{d,\sigma} - \bar{\gamma}_\sigma I & 0 \\ * & 0 & -\bar{\gamma}_\sigma I \end{pmatrix} \preceq 0 \tag{20}$$

$$\begin{pmatrix} \Pi_{d,\sigma} & \Phi_{d,\sigma} & P_\sigma \tilde{I} \\ * & -\bar{\gamma}_\sigma I & 0 \\ * & 0 & -\bar{\gamma}_\sigma I \end{pmatrix} \preceq 0 \tag{21}$$

$$\begin{pmatrix} -\bar{\alpha}_\sigma I & P_\sigma \\ P_\sigma & -I \end{pmatrix} \succeq 0 \tag{22}$$

$$P_\sigma \succeq I \tag{23}$$

where

$$\begin{aligned} \Pi_{d,\sigma} &= \tilde{A}_\sigma^T P_\sigma + \tilde{C}_\sigma^T U_\sigma + P_\sigma \tilde{A}_\sigma + U_\sigma \tilde{C}_\sigma + I \\ \Phi_{d,\sigma} &= P_\sigma \tilde{E}_{d,\sigma} + U_\sigma F_{d,\sigma} \\ \Omega_{d,\sigma} &= \tilde{A}_\sigma^T P_\sigma + \tilde{C}_\sigma^T U_\sigma + P_\sigma \tilde{A}_\sigma + U_\sigma \tilde{C}_\sigma + \tilde{C}_\sigma^T \tilde{C}_\sigma \\ \Lambda_{d,\sigma} &= P_\sigma \tilde{E}_{d,\sigma} + U_\sigma F_{d,\sigma} + \tilde{C}_\sigma F_{d,\sigma} \end{aligned}$$

then it is ensured that

$$\|\tilde{x}\|_2 \leq \sqrt{\frac{\lambda_{\max}(P_\sigma)}{\lambda_{\min}(P_\sigma)}}\left(e^{-\frac{t}{2\lambda_{\max}(P_\sigma)}}\|\tilde{x}(0)\|_2 + \gamma_\sigma \left\|\tilde{d}\right\|_\infty\right)$$

$\gamma_\sigma = \sqrt{\bar{\gamma}_\sigma}$ and $\beta_\sigma \in [0, 1]$ is a design parameter that balances between the two objectives of minimizing $\bar{\alpha}_\sigma$ and $\bar{\gamma}_\sigma$. The LMI condition (20) ensures the H_∞ attenuation, while the LMI conditions (21) to (23) ensure the ISS property of the designed observers. The LMI condition (23) ensures that $\lambda_{\max}(P_\sigma) \leq \alpha_\sigma$, $\alpha_\sigma^2 = \bar{\alpha}_\sigma$ since $\lambda_{\min}(P_\sigma) \geq 1$.

The robust PI fault detection observer is with gain filter $L_\sigma = -P_\sigma^{-1} U_\sigma$ and $\left\| r_\sigma|_{f=0} \right\|_2 < \gamma_\sigma \left\| [d^T \xi^T]^T \right\|_2$. The solution of the LMI problem is conducted under available software.

The existence of the proposed PI observer is subject to the pair $\left(\tilde{A}_\sigma, \tilde{C}_\sigma\right)$ is observable or at least detectable. In addition, in order to be able to estimate the fault vector $f(t)$, the following rank condition should hold for all $s \in \mathbb{C}$ such that $\Re s > 0$ and $\sigma = 1, 2, ..., N$:

$$rank \begin{bmatrix} sI - A_\sigma & -E_{f,\sigma} \\ 0 & sI \\ C & 0 \end{bmatrix} = n_x + rank\left(E_{f,\sigma}\right) \qquad (24)$$

Since the speed variations are not very fast, there is an average dwell time of each mode which ensures stability [19]. Another approach is to add pole location constraint [16]. Due to space limitation, the study has not been carried out.

IV. SIMULATION STUDIES

The observability is ensured if at least two measurements are used. However, when aiming to estimate both the steering torque and the roll angle, the measurement of the steering angle and the yaw rate are required. In the following, it is assumed that the measurement vector is $y = \left[\delta, \dot{\psi}, \dot{\phi}\right]^T$.

The speed range $[10, 30]$ m/s is discretised by steps of 5 m/s. An observer gain is computed for each subsystem. The observers are then combined according to speed variations.

The chosen scenario for simulation is a double lane change manoeuvre a typical situation when the rider avoids an obstacle located on the lane. The action is a positive torque on the handlebars followed by a negative torque. This manoeuvre excites the motorcycle dynamics, the corresponding torque and longitudinal speed variations are shown in Figure 1. The longitudinal speed profile starts at 12 $[m/s]$. It increases until 20 $[m/s]$, stays constant during 4 sec and then decreases to 10 $[m/s]$. Doing that and according to speed discretisation, three subsystems in the speed switched system are involved.

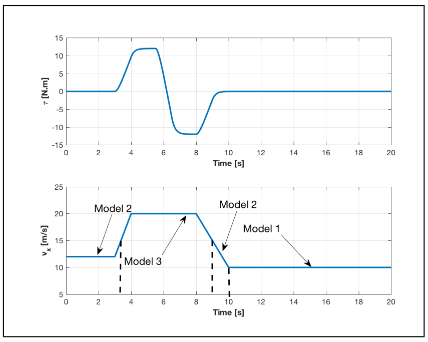

Fig. 1. Rider torque and speed profile during double lane change maneuver

In all the simulation, the initial conditions of the observers are chosen different from that of the system in order to shown the behaviour before convergence. First of all, observers are developed with the motorcycle roll angle and the rider torque considered faults to be estimated, parameters variations enter as unknown input.

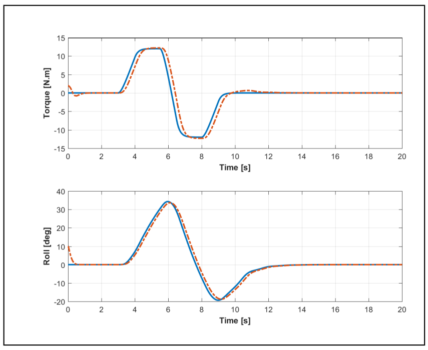

Fig. 2. Rider torque and roll angle estimation. Estimate in dash-dot lines.

The results are shown in Figure 2. One can see that the estimation of both variables works well. The estimated variables follow the variation of the roll and the handlebars torque. The convergence time of the observer is sufficiently short. The noisy case is now treated. Figure 3 shows the three noisy measured variables. The estimated tire forces are shown in Figure 4. According to the maximum values, the saturation zone is reached and the estimation still works.

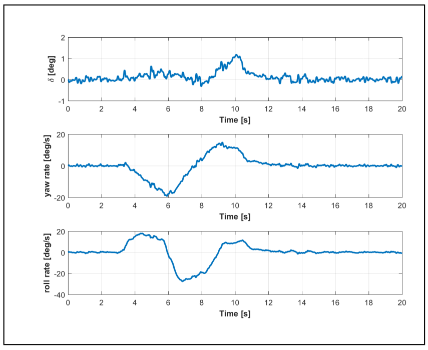

Fig. 3. Noisy measurements $\left[\delta, \dot{\psi}, \dot{\phi}\right]^T$

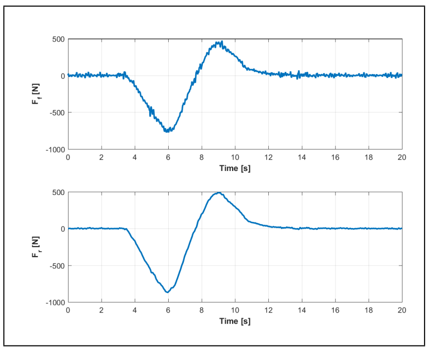

Fig. 4. Estimated lateral forces $\hat{F}_f$ and $\hat{F}_r$

The estimated roll angle and rider torque in this conditions are shown in Figure 5. The obtain profiles do not exhibit any

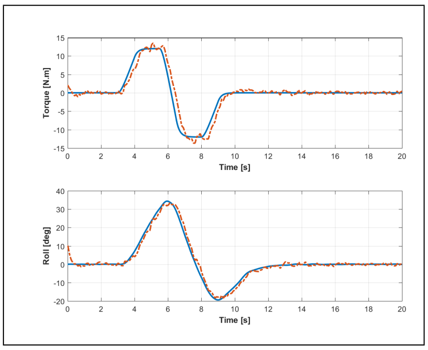

Fig. 5. Rider torque and roll angle estimation in noisy conditions. Estimates in dash-dot lines.

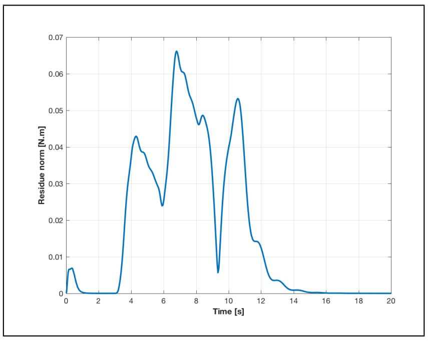

Fig. 6. Residual signal

bias and the delay is small. The observer reacts well to the variations. Figure 6 shows the norm of the residue $r_\sigma(t)$. The obtained values are very small.

V. CONCLUSIONS

In this paper a robust fault estimation framework for non-linear switched systems has been presented. It uses a switched H_∞ PI observer structure. The observers allow the robust estimation of the fault while minimizing the effect of the unknown disturbance input on the residue. The observer gains are obtained from the solutions of an optimization problem under LMI conditions which include ISS property. This problem is solved using efficient available LMI solver.

This framework is applied to the estimation of unknown input and non-linear behaviour in motorcycle's lateral dynamics. The motorcycle is subject to disturbance inputs ans the tire forces may experience saturations that render it less controllable. The observers provide a robust estimation of the gap between the normal linear behaviour of the forces and the experienced non-linear behaviour. Prior to the observer synthesis an observability analysis is conducted together with the sensors requirements.

A motorcycle obstacle avoidance manoeuvre is simulated in order to evaluate the capabilities of the proposed observers. Future work will concern first further analysis of the design procedure including the simultaneous use of a bank of dedicated observers. The development and the implementation of a system with provides assistance to the rider will concern the second aspect of future research. It will use visual and haptic feedback.

APPENDIX

The motorcycle model parameters are as follows:

- Mass: Front body $(M_f = 30.26)$, rear body $(M_r = 217.45kg)$, motorcycle $(M = M_f + Mr)$.
- Geometrical parameters $l_f = 0.935m$, $l_r = 0.480m$, $h = 0.615m$, $\varepsilon = 0.4712rad$, .
- Stiffness $c_{f1} = 11.174kN/rad$, $c_{f2} = 0.9386kN/rad$, $c_{r1} = 15.8312kN/rad$, $c_{r2} = 1.3256KN/rad$, $\eta = 6.780N.m.s/rad$.
- $R_f = 0.3048m$, $i_{fy} = 1.051kg.m^2$.

Other parameters are available in [18].

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