Switched interval observer for uncertain continuous-time systems

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Abstract: This paper addresses the problem of robust state estimation of switched uncertain systems subject to unknown disturbances. The proposed approach is based on switched interval observers which provide guaranteed lower and upper bounds allowing to evaluate the set of admissible values of the real state vector. The stability and cooperativity conditions of the proposed switched observer are expressed in terms of linear matrix inequalities (LMIs), witch have been established using a common quadratic Lyapunov function (CQLF). Estimation accuracy and robustness with respect to unknown disturbances is analyzed using $\mathcal{H}_\infty$ objective with pole placement constraints. The proposed approach is illustrated by a numerical example.

Keywords: Interval observer, Switched systems, Switched positive systems, Uniform asymptotic stability, Quadratic common Lyapunov function, H-infinity.

1. INTRODUCTION

In recent years, switched systems gained a great attention due to their wide applications in practical systems. Switched linear systems (Liberzon (2012)) are an important class of hybrid dynamic systems (Goebel et al. (2009)) consisting of a several subsystems and a switching discrete law specifying at each time the active subsystem dynamics. Contrary to the stability problem of switched systems that has been extensively studied in the literature (see for example Liberzon and Morse (1999), Agrachev and Liberzon (2001), Vu et al. (2007), Agrachev and Liberzon (1999)), the estimation problem for this class of systems has attracted less attention and very few works are available in this area, (see Li et al. (2003), Alessandri and Coletta (2001)).

The state estimation problem of switched systems was originally studied in Ackerson and Fu (1970), afterward, Alessandri and Coletta proposed a optimally switching Luenberger observer for continuous-time deterministic linear switched systems with a known discrete law evolution (see Alessandri and Coletta (2001)). In Balluchi et al. (2002), this approach has been extended to the case of unknown discrete modes and a method of discrete and continuous state estimation for linear systems has been presented.

Notice that all the mentioned approaches are based on a perfect knowledge of the model structure and parameters. However, the uncertainties can generate a large bias in the estimation of unmeasured states. Consequently, we will focus here on the problem of robust estimation of switched uncertain systems taking into account the presence of unknown disturbances with the potential to evaluate the set of admissible trajectory of systems. Using the so-called interval observers, we will be able to provide guaranteed bounds of the real trajectories.

Interval observers are appeared in last decade as an alternative approach for robust estimation (Rapaport and Harmand (2002)). They were originally developed in Gouzé et al. (2000) for the estimation of biological systems subject to unknown uncertainties. These observers require, in addition to stability, the cooperativity and positivity of observation error (Smith (2008)). There are various approaches to design interval observers for continuous times systems satisfying properties of monotone differential systems. See for instance, Rami et al. (2008), Bolajraf et al. (2010) and Rami et al. (2013), where interval observers for linear uncertain systems are presented. The necessary and sufficient conditions have been formulated in terms of linear programming. In Raïssi et al. (2010), Efimov et al. (2012) and Efimov et al. (2013), the interval observers for LPV and nonlinear systems based on Lyapunov theory and linear matrix inequalities (LMIs) have been designed.

The objective of this technical note is to propose some new results on interval observer for switched uncertain linear systems with guaranteed of cooperativity and uniform asymptotic stability of the switched observers.

The paper is organized as follows. In Section 2, we present the preliminaries and the mathematical background necessary to state the main result of the paper. The necessary and sufficient conditions for the existing of switched interval observer are presented in section 3. Section 4 contains an illustrative example. Finally, the section 5 concludes the paper.

Notations: Throughout the paper, we will adopt the convention of denoting a left and right endpoints of an interval $[x]$ respectively by x^- and x^+ such as $[x] = [x^-, x^+]$. For any two vectors x_1, x_2 or matrices M_1, M_2 the inequalities $x_1 \leq x_2$, $x_1 \geq x_2$, $M_1 \leq M_2$ and $M_1 \geq M_2$ must be interpreted element-wise. $m_{\hat{i}\hat{j}}$ denotes the element on the

$\hat{i}$th line and $\hat{j}$th column of the matrix M. $M > 0$ (resp. $M < 0$) denotes a matrix with positive (resp. negative) components and $M \succ 0$ (resp. $M \prec 0$) means that the matrix is positive (resp. negative) semidefinite. M^T means the transpose of matrix M. $\mathbb{R}$ ($\mathbb{R}_+$) is the set of all real (positive) numbers. $\mathbb{R}^n$ ($\mathbb{R}^n_+$) is n-dimensional real (positive) vector space. We denote by $\mathcal{I}_n$ an identity matrix of dimension $n \times n$.

2. BACKGROUND AND PRELIMINARIES

In this paper, we will be interested by robust estimation of the linear switched systems of the form:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + \omega(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$, represent respectively the state, the control input and the output vector. $\sigma : \mathbb{R}^+ \rightarrow \{1, 2, ..., N\}$ is a piecewise constant function representing the switching signal, which are assumed to be available in real time. The matrices $A_{\sigma(t)} \in \{A_1, A_2, ..., A_N\}$ and $B_{\sigma(t)} \in \{B_1, B_2, ..., B_N\}$ are assumed to be known. Without loss of generality, the output matrix C is considered constant $\forall\sigma(t)$. The disturbance input $\omega(t)$ is a Lipschitz continuous function, bounded by two known functions such that $\forall t \geq 0$, $\omega^-(t) \leq \omega(t) \leq \omega^+(t)$.

The objective of this work is to design a switching interval observer for the system (1). The designed estimator has to ensure a robustness with respect to unknown perturbation $\omega(t)$. The following lemmas and definitions will be used later to establish the main result.

Interval observers and Metzler matrices

Definition 1. (Rami et al. (2008)) An interval observer for (1) is a pair of estimators providing a lower and upper bounds (x^- and x^+) of the real state $x(t)$ such that $x^-(t) \leq x(t) \leq x^+(t)$, $\forall t \geq t_0$.

Definition 2. A real matrix A. is called a Metzler matrix if all its elements outside the main diagonal are positive: $a_{\hat{i}\hat{j}} \geq 0, \ \forall \hat{i} \neq \hat{j}$.

Lemma 1. (Rami et al. (2008)) A matrix A_i is a Metzler if and only if there exist $\beta \in \mathbb{R}_+$ such that $A_i + \beta\mathcal{I}_n \succeq 0$.

Cooperative and Positive switched systems

Definition 3. The continuous-time switched system (1) is said to be cooperative if the state matrix A_i is Metzler $\forall i \in \{1, ..., N\}$ and $\forall t \geq t_0$.

Remark 1. Definition 3 is also equivalent to saying that the system (1) is monotone. This means that given any two initial conditions $x_A(t_0)$ and $x_B(t_0)$ such that $x_A(t_0) \leq x_B(t_0)$, the corresponding solutions $x_A(t)$ and $x_B(t)$ satisfy $x_A(t) \leq x_B(t)$, $\forall t \geq t_0$.

Definition 4. The switched system (1) is said to be a positive switched system (Blanchini et al. (2015)) if A_i is a $n \times n$ Metzler matrix, $B_i \geq 0, \forall i \in \{1, ..., N\}$, $u(t) \geq 0$ and $\omega(t) \geq 0$ and $\forall t \geq t_0$.

This ensures that $\forall x_0 \in \mathbb{R}^n_+$, and for every choice of σ, the state evolution $x(t) = x(t; x_0, \sigma)$ belongs to $\mathbb{R}^n_+$ for every $t \geq t_0$.

Using these properties (Cooperativity and positivity), the interval switched observers will keep the partial order between the lower and upper trajectories.

Uniform asymptotic stability of switched positive systems

In this paper, we address the uniform asymptotic stability. This issue is usually used when there is no restriction on the switching signals and requires that all the subsystems are asymptotically stable.
Note that the subsystems stability assumption is not sufficient to guaranteed the stability of the switched system under arbitrary switching. Therefore, it has been shown that if there exists a common quadratic Lyapunov function (CQLF) for all the subsystems, then the stability of the switched system is insured under arbitrary switching (Shorten and Narendra (1998)). The word "*Uniform*" refers to uniformity with respect to switching signals. Formally, checking for the existence of a CQLF can be expressed in terms of linear matrix inequalities (LMIs) as follows:

Lemma 2. If there exists a common positive definite matrix $P = P^T \succeq 0$ in $R^{n\times n}$ satisfying the N Lyapunov inequalities:

$$A_i^T P + PA_i \prec 0 \quad i \in \{1, ..., N\} \tag{2}$$

then $V(x) = x^T Px$ defines a common quadratic Lyapunov function for the systems of the form (1).

Pole assignment problem in LMI region

Consider the LMI region given by the stability margin α_i (Figure 1). The matrix A_i is said $\mathcal{D}_i$-Stable when its spectrum $\lambda(A_i)$ belongs to region $\mathcal{D}_i$:

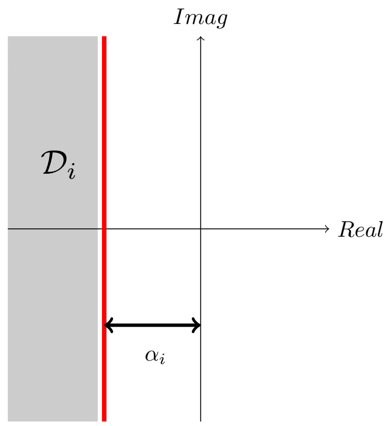

Fig. 1. Pole placement in region LMI for the subsystem i.

$$\mathcal{D}_i = \{z \in \mathbb{C} \ / \ Re\,(z) < -\alpha_i\}$$

This is equivalent to the following Lyapunov inequalities:

$$A_i^T P + PA_i + \alpha_i P \prec 0 \quad i \in \{1, ..., N\} \tag{3}$$

with $\alpha_i > 0$ the desired stability margin.

3. SWITCHING INTERVAL OBSERVER DESIGN FOR SWITCHED LINEAR SYSTEMS

To design a switching stable interval observer for the system (1), the choice of the observer gains L_i such that $A_i - L_iC$ is only Hurwitz is not sufficient. Therefore, additional assumptions are used to prove the positivity of observation errors (Smith (2008)).
In this section, we present a construction method of switching interval observers for the system (1). We introduce some assumptions required for our design.

Assumption 1. The pair (A_i, C) is detectable $\forall i \in \{1, ..., N\}$.

Assumption 2. There exist $\omega^-(t)$, $\omega^+(t)$ such that:

$$\omega^-(t) \leq \omega(t) \leq \omega^+(t) \tag{4}$$

Under assumptions 1 and 2, the following proposed system:

$$\begin{cases} \dot{x}^+(t) = (A_{\sigma(t)} - L_{\sigma(t)}C)x^+ + B_{\sigma(t)}u + L_{\sigma(t)}y + \omega^+(t) \\ \dot{x}^-(t) = (A_{\sigma(t)} - L_{\sigma(t)}C)x^- + B_{\sigma(t)}u + L_{\sigma(t)}y + \omega^-(t) \\ x^-(t_0) \leq x(t) \leq x^+(t_0) \end{cases} \tag{5}$$

is a switching interval observer for the system (1) if:

$$x^-(t) \leq x(t) \leq x^+(t), \quad \forall t \geq t_0 \tag{6}$$

3.1 Positivity of the interval errors

The inequality (6) is equivalent to showing that the upper and lower errors $e^-(t) = x(t) - x^-(t)$ and $e^+(t) = x^+(t) - x(t)$ are positive for all initial conditions $e^-(t_0) = x(t_0) - x^-(t_0) \geq 0$ and $e^+(t_0) = x^+(t_0) - x(t_0) \geq 0$ and it suffices to use the lemma 1 to show the nonnegativity of the observation errors. Their dynamics are given by:

$$\begin{cases} \dot{e}^+(t) = (A_{\sigma(t)} - L_{\sigma(t)}C)e^+(t) + \delta^+(t) \\ \dot{e}^-(t) = (A_{\sigma(t)} - L_{\sigma(t)}C)e^-(t) + \delta^-(t) \end{cases} \tag{7}$$

with

$$\begin{cases} \delta^+(t) = \omega^+(t) - \omega(t) \\ \delta^-(t) = \omega(t) - \omega^-(t) \end{cases} \tag{8}$$

The input $\delta^+(t)$ and $\delta^-(t)$ are nonnegative for all $t \geq t_0$ due to assumption 2 then according to definition 4, the estimation errors are positive for all $e^-(t_0) \geq 0$ and $e^+(t_0) \geq 0$ if and only if the matrix $A_i - L_iC$ is Metzler $\forall i$. Positivity of the interval errors is necessary to satisfy the interval property, i.e, the state trajectories stay within the estimated bounds.

3.2 Convergence of interval error

Note that the stability of the proposed switching interval observer results from the stability of total interval error $e(t) = x^+(t) - x^-(t)$ witch represents the difference between the upper and lower state estimates.
The dynamic of the total error $e(t)$ is given as follows:

$$\dot{e}(t) = (A_{\sigma(t)} - L_{\sigma(t)}C)e(t) + \delta(t) \tag{9}$$

where

$$\delta(t) = \delta^+(t) - \delta^-(t)$$

The dynamics of total error behaves like a switched system, therefore a common quadratic Lyapunov function is searched to ensure stability.

Thus, the main result is given by the following theorem

Theorem 1. If there exist a positive definite diagonal matrix P, real positive scalar $\overline{\gamma}$ and matrices K_i satisfying:

$$\min_{P,K_i} \overline{\gamma}$$

$$\begin{bmatrix} A_i^TP - C_i^TK_i^T + PA_i - K_iC_i + \alpha_iP & P \\ P & -\overline{\gamma}\mathcal{I}_n \end{bmatrix} \prec 0 \tag{10}$$

$$PA_i - K_iC_i + \beta P \geq 0$$

with a given positive constant β and $L_i = P^{-1}K_i$, then the switched interval observer (5) involves a positive estimation error such that

(1) The total estimation error (9) with $\delta(t) = 0$ is uniform asymptotically stable.

(2) The estimation error $e(t)$ is uniformly bounded and satisfies

$$sup \ \frac{\|e(t)\|_2}{\|\delta(t)\|_2} \leq \gamma \tag{11}$$

for all bounded $\delta(t) \neq 0$, where $\gamma = \sqrt{\overline{\gamma}}$ represents the disturbance attenuation level.

Proof. In order to improve the performance of the switching interval observer, we define a variable α_i such that all the eigenvalues of $(A_i - L_iC)$ lie inside a specific LMI region $\mathcal{D}_i$, $\forall i \in \{1, ..., N\}$ (See figure 1). Then, for a common Lyapunov function $V(t) = e(t)^TPe(t)$, the time derivative should satisfy

$$\dot{V}(t) < -\alpha_iV(t) \tag{12}$$

Along the trajectory of each mode i of the observation error equation given in (9), we have

$$\dot{V} = e^T((A_i - L_iC)^TP + P(A_i - L_iC))e + \delta^TPe + e^TP\delta \tag{13}$$

it follows from (12) and (13) that

$$e^T((A_i - L_iC)^TP + P(A_i - L_iC))e + \delta^TPe + e^TP\delta \prec -\alpha_ie^TPe \tag{14}$$

then, the total error $e(t)$ given in (9) is $\mathcal{D}_i$-Stable if the following inequality hold for each mode i

$$e^T((A_i - L_iC)^TP + P(A_i - L_iC) + \alpha_iP)e + \delta^TPe + e^TP\delta \prec 0 \tag{15}$$

In the other hand, we want to minimize the influences of the disturbances $\delta(t) \to e(t)$ according to the $\mathcal{H}_\infty$ criteria, with index γ such that:

$$\|e\|_2 < \gamma\|\delta\|_2 \tag{16}$$

equivalent to find $\gamma \in \mathbb{R}_+$ satisfying

$$e^Te - \gamma^2\delta^T\delta \prec 0 \tag{17}$$

Then according to the constraints formulated in (15-17) and by using the $\mathcal{S}$-procedure (Chen et al. (2000)), we obtain the equivalent inequalities:

$$e^T(A_i^T - C^T L_i^T + PA_i - PL_iC + \alpha_i P)e + \delta^T Pe + e^T P\delta$$
$$-\gamma^2\delta^T\delta \prec 0 \tag{18}$$

for each $i \in \{1, \dots, N\}$. Let $K_i = PL_i$ and $\overline{\gamma} = \gamma^2$, then we get the following linear matrix inequalities:

$$\min_{P,K_i} \ \overline{\gamma}$$

$$\begin{bmatrix} A_i^T P - C_i^T K_i^T + PA_i - K_iC_i + \alpha_i P & P \\ P & -\overline{\gamma}\,\mathcal{I}_n \end{bmatrix} \prec 0 \tag{19}$$

The previously constraints ensure that $(A_i - L_iC)$ is Hurwitz $\forall i$. In a second step, we need to ensure the Metzler property. Referring to lemma 1, $(A_i - L_iC)$ is Metzler if:

$$(A_i - L_iC) + \beta\mathcal{I}_n \geq 0 \tag{20}$$

Note that the quantity $\beta\mathcal{I}_n$ is added in the Metzler constraint since only the off-diagonal elements of a matrix must be nonnegative to satisfy the Metzler property. Furthermore for a given positive definite diagonal matrix P, the matrix $P(A_i - L_iC)$ is also Metzler $\forall i$ (Chebotarev et al. (2015)). Multiplying by P in the left side of (20) and developing, we obtain:

$$PA_i - K_iC_i + \beta P \geq 0 \tag{21}$$

which proves the positivity of the interval error e for a diagonal positive definite matrix P thus we conclude that the observation error e is always positive-stable and the proof is complete. $\square$

Remark 3. Note that the previous inequality is not linear for the parameters β and P because of the presence of the product βP. However, by fixing the scalar β, the Bilinear Matrix Inequality (BMI) described by equations (21) is transformed into Linear Matrix Inequality form.

4. ILLUSTRATIVE EXAMPLE

In this section, we apply the switched interval observer previously described to estimate the state vector of a two-degree-of-freedom switched system with two modes given as follows

Subsystem 1 $(\sigma(t) = 1)$

$$S_1 : \begin{cases} \dot{x}(t) = \begin{bmatrix} -1 & 0.1 \\ -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \omega(t) \\ y(t) = [1 \quad 0]\, x(t) \end{cases} \tag{22}$$

Subsystem 2 $(\sigma(t) = 2)$

$$S_2 : \begin{cases} \dot{x}(t) = \begin{bmatrix} -1 & 0.5 \\ -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) + \omega(t) \\ y(t) = [1 \quad 0]\, x(t) \end{cases} \tag{23}$$

with $u(t) = sin(t)$ and $\omega(t)$ an uniformly distributed noise in $[-1, 1]$. The switching law is presented in figure 2.
Solving the linear matrix inequalities in (10) give the solutions

$$L_1 = \begin{bmatrix} 12.3391 \\ -6.7799 \end{bmatrix} \qquad L_2 = \begin{bmatrix} 14.9755 \\ -5.6590 \end{bmatrix}$$

and

$$P = \begin{bmatrix} 0.0084 & 0 \\ 0 & 0.0012 \end{bmatrix}$$

$\gamma = 0.0316$. The numerical simulation was carried out by using Matlab YALMIP toolbox. The numerical results corresponding to an initial conditions $x_0^+ = [0.2\ 0.2]^T$, $x_0^- = [-0.2\ -0.2]^T$, $\beta = 1$, a stability margins $\alpha_1 = \alpha_2 = 2$, $\omega^+(t) = \omega(t) + 0.2$ and $\omega^-(t) = \omega(t) - 0.2$.

The switched interval observer for x_1 and x_2 is shown in figures 3 and 4. The simulation of proposed observer proves the effectiveness of such observer to estimate guaranteed bounds of state vector. The evolution of the interval errors $e_1^+(t) = x_1^+(t) - x_1^-(t)$ and $e_2^-(t) = x_2^+(t) - x_2^-(t)$ are depicted in figure 5. In fact, the interval errors dynamics converge in a finite time towards an invariant set defined by the disturbance bounds $\omega^+(t)$ ans $\omega^-(t)$ and calculated attenuation gain γ. Note that, if $\omega(t) = 0$, then the proposed observer will converge asymptotically to zero.

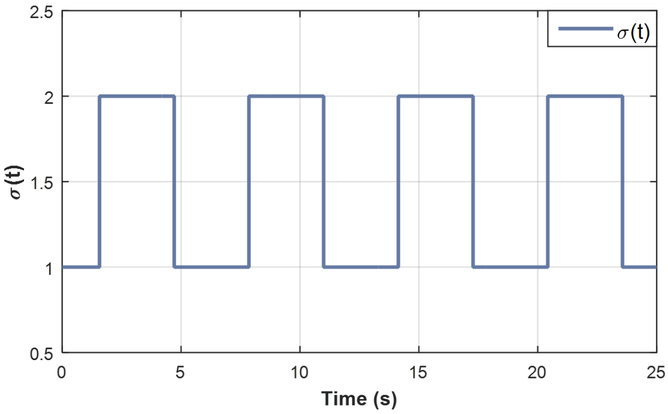

Fig. 2. Switching signal.

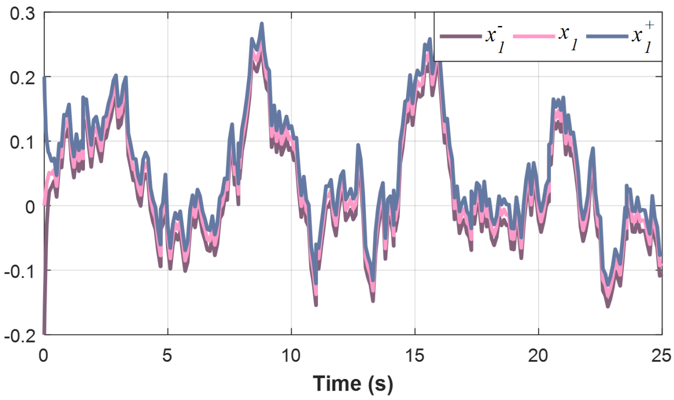

Fig. 3. Switching interval observer of the state x_1.

5. CONCLUSION AND FUTURE WORKS

In this paper, we have addressed the design problem of a switched interval observer for a class of switched uncertain continuous-time linear systems subject to exogenous disturbances. The necessary and sufficient conditions ensuring the positivity and stability of the interval error dynamic have been found and an LMI formulation has

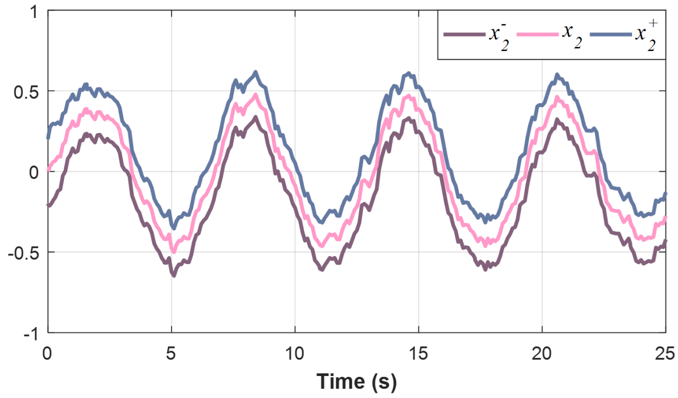

Fig. 4. Switching interval observer of the state x_2.

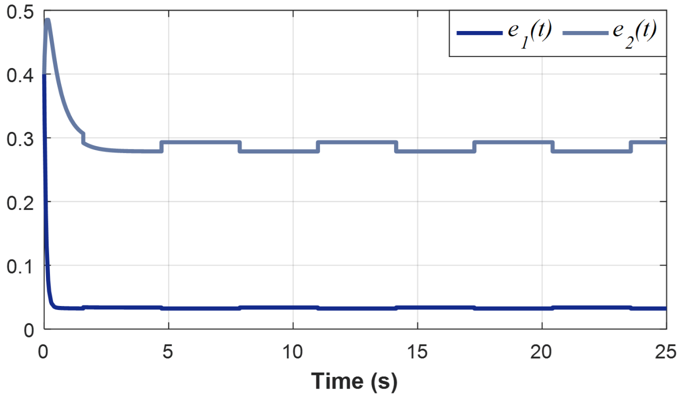

Fig. 5. Interval errors $e_1(t)$ and $e_2(t)$.

been presented to compute the switched optimal observer gains. The simulation results obtained for a simple two-degree-of-freedom system subject to additive unknown and bounded disturbances, show both that, the proposed estimators are stable for arbitrary switching and that they ensure guaranteed bounds with a higher performance.

Future works will concern the extension of the proposed interval observer for the linear and nonlinear switched systems with uncertain parameters and measurement noises. Reduction of the conservatism of the proposed LMIs using a multiple quadratic Lyapunov function will be also considered.

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