Lateral & Steering Dynamics Estimation for Single Track Vehicle: Experimental Tests ⋆

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Abstract: This paper deals with lateral and steering dynamics estimation of powered two-wheeled (PTW) vehicles. It presents an experimental validation of the Unknown Input Observer (UIO) addressed in Damon et al. (2016). A small scooter equipped with a multi-sensor architecture used to performed the test is introduced. A mathematical model of the scooter is derived using measured parameters on a Computer-Aided Design (CAD) model. Then the main design steps of the UIO are shortly remind. Finally, an urban driving scenario is presented to show the effectiveness of the proposed observer to estimate the lateral dynamics and the rider's action in real riding scenario.

Keywords: Experimentation, Motorcycle, Observation, Linear Parameter Varying (LPV) Systems, Unknown Input Estimation.

1. INTRODUCTION AND MOTIVATIONS

Nowadays, more and more vehicles are equipped with Advanced Driver Assistance Systems (ADAS). These systems are included in a more general topic which is Intelligent Transportation Systems (ITS). ITS is the application of the new technologies on transportation field including vehicle, environment and infrastructure. Since autonomous driving and road user's safety became real challenges there is a high interest for ADAS. They have to realize particular tasks such as: vehicle localization, automatic guidance, obstacle avoidance, pedestrian detection, stability control, etc. Most of them are dedicated to improve safety by informing the driver about dangerous situations and sometimes by acting on vehicle dynamics. However, ADAS developed during last years were mainly aimed to automotive industry. Indeed, Powered Two-Wheeled (PWT) market is cheaper, it is difficult to cover instrumentation (sensors, embedded electronic, ECU, etc.) and R&D costs with an attractive selling price. Moreover, most of these systems are based on a mathematical models of the vehicle, and motorcycles are much more complex to model and strongly nonlinear than four-wheeled vehicles.

Nevertheless, since several motorcycle models have been proposed Sharp (1971), Sharp et al. (2004), Cossalter (2006), Pacejka (2006) and Nehaoua et al. (2013a), embedded electronics is becoming common and sensors are affordable. ADAS for motorcycle, recently, becomes an essential research issue for motorcycle manufacturer. Their performance highly depends of the embedded architecture including global electronic architecture, sensor integration and limitation, sampling frequency, etc. Such architectures allow to facilitate the implementation of algorithms by managing the communication between algorithms and sensors. For instance, they can collect sensor data and control the system without blocking algorithm execution. The topic of electronic architecture was largely addressed for four-wheel vehicles but few architectures exist for PWT. This paper proposed a solution of architecture for experimental investigation.

Today several systems exist on motorcycle market: Antilock Braking System (ABS), Traction Control System (TCS), Motorcycle Stability Control (MSC), etc. But for most of them they equipped premium motorcycles which represent a tiny proportion of the whole of motorcycle park. Estimation and observation are major tools to make easier the development of ADAS by allowing a reduction of the number of sensors and hence reducing the cost. It could be the best solution to make it available on a large proportion of sold motorcycles. Lot of recent works deal with motorcycle dynamic state estimation Dabladji et al. (2015), Nehaoua et al. (2013b) and Ichalal et al. (2013) but few of them perform experimental investigations to validate the results. In Dabladji et al. (2016) or in Filippi et al. (2010) authors proposed a validation on BikeSim which is a multi-body motorcycle simulator. Even if such simulator is known for its ability to simulate real scenarios, it is difficult to take into account the inherent problems related to real tests like sensor noises or faults. This type of simulator can give a first idea about observer performance but cannot replace real implementations. In Boniolo et al. (2012) and Lot et al. (2012) experimental tests are performed but only roll angle estimation is considered. In Teerhuis and Jansen (2012) authors proposed a more complete experimental investigation to validate estimated states with extended Kalman filter but only medium speed are considered and speed range is very reduced.

⋆ This work is supported by National Agency of Research under the framework VIROLO++.

The main contribution of this paper is the experimental validation of the proposed works in Damon et al. (2016) by designing the observer on a simple motorcycle model. Indeed, the next objective is to estimate dynamic states in-line which means while the vehicle is moving and detect in real-time critical situations. That is why the observer design is based on a simple two bodies model of the scooter which allows to get very good estimation time performance. Let us remind that the considered UIO is able to simultaneously estimate every lateral dynamic states and rider's action without any forward speed limitation in contrast with Teerhuis and Jansen (2012) or in Ichalal et al. (2013).

This paper is organized as follows: Section 2 introduces the vehicle and the embedded architecture used to perform the tests. Section 3 explains how the mathematical model of the vehicle is derived. Section 4 aims to remind the main steps in the observer design presented in Damon et al. (2016). Section 5 discusses the results of the experimental tests. Finally, section 6 presents concluding remarks and discussions.

2. VEHICLE AND SENSOR ARCHITECTURE

2.1 Instrumented scooter

The instrumented PWT vehicle in order to perform the tests is a Peugeot Scoot'elec (fig. 1). It is equipped with an electric power-train developing a maximum power around 3 kW allowing a maximum speed of 45 km/h. Three blocs of NiCad batteries with, for each one, a capacity of 100 Ah for 6 V voltage feed the motor and inevitably lead to a consequent weight of 115 kg without rider. The vehicle can reach a maximum distance between 40 and 60 km depending on the driving behavior with a full electric charge. An additional top case is fixed at the rear of the scooter to embed computer and data acquisition devices.

2.2 Embedded computer

In order to perform the various calculations and dynamic variable measurements, we have chosen a computer dedicated to embedded applications manufactured by Neousys Technology (fig. 1). According to its compact size the model NUVO-3005EB is ideal for installation under a seat or in a top case and offers several features and benefits like: High performance GPU (Intel Core i7-3610QM), PCIe Expansion Slot, Wi-Fi, 3G and GPS options integrated.

2.3 Digital/Analog IO interface card

Regarding the number of sensors, their features (operating range, analog/digital, maximum sampling frequency, etc.), we have opted for the PCIe-6353 card manufactured by National Instrument. This version is able to achieve sampling rate until 10 MHz/s and is provided with a 68 pin box referenced by SCB-68. This is an extension board for direct connection of sensors. This solution allows the use of different software for data acquisition, one can find dedicated tool boxes in Labview or in Matlab/Simulink. The second solution have been considered to perform the tests given below.

Fig. 1. Instrumented Scooter

2.4 Inertial Measurement Unit

The Inertial Measurement Unit (IMU) is a IG-500A manufactured by SBG Systems (fig. 1). It can work on angular movements of 360° on the 3 axes and offers orientation matrix either on Cartesian or Euler angles. The needed measures for our application are:

- The angular positions: roll, pitch and yaw respectively ϕ, γ and ψ
- The angular rates: roll rate, pitch rate and yaw rate respectively $\dot{\phi}$, $\dot{\gamma}$ and $\dot{\psi}$
- The axial accelerations: longitudinal, lateral and vertical acceleration respectively a_x, a_y and a_z in a local frame

The IMU is placed as close as possible to the gravity center of the whole driver and scooter (fig. 1) and its orientation is based on the theoretical referential used in the Sharp model of the motorcycle. Therefore, it directly sends the vehicle axial accelerations, angle and rotational speeds data around each axis (the rotational motion of the earth is assumed to be negligible).

2.5 Steering angle encoder

Several methods are possible in instrumentation of steering mechanism. We made the choice of an absolute encoder to measure the handlebar position. Several architectures are possible, directly installed on the steering column, the encoder gives the steering angle δ without any transformation or ratio or another possibility is proposed in Mammar et al. (2006) where authors have chosen the use of a pulley system with a belt linked to the steering column. The first solution is considered because it is simpler to implement and gives direct information about the front wheel angle relatively to the frame. The selected sensor is an absolute encoder of GA210 type manufactured by IVO Industries (fig. 1), it is a 10 bit resolution, 1024 steps per turn with parallel output and a 400 Hz sampling frequency.

2.6 Wheel rotation speed sensor

Mostly incremental encoders are used to make a measurement for a distance and a speed. As for production vehicles, whether motorcycle or automobile manufacturers, they use single pulse Hall effect sensors. For our architecture, optical encoder has been chosen for the measurement of the wheel rotation speed. The optical encoder is a KTIR 0221 DS , manufactured by Kingbright (fig. 1).

The measure will be expressed in number of counted teeth, knowing the number of teeth per revolution the rotational speed is easy to obtain. Deduct the forward speed v_x from the wheel velocity ω is not trivial because the speed depends of the longitudinal slip of both wheels as follows:

$$\lambda_i = \frac{v_x - R_i\omega_i}{max(v_x, R_i\omega_i)} \quad i = f, r \tag{1}$$

On our scooter the rear Hall effect sensor is installed on the belt receiving pulley which has 8 teeth. This pulley transfers the motor torque and speed to the rear wheel through a fixed gear ratio of 13/47.

3. MATHEMATICAL MODEL OF THE SCOOTER

Over the last 50 years modeling motorcycle was a real challenge and lot of literature have been provided about motorcycle modeling. One main difference between the proposed models is the complexity due to the number of bodies with whom the motorcycle is modeled. One can find a single body model in van Daal (2009) which is a simple inverse pendulum or very complicated one as in Sharp et al. (2004) where the whole PWT and rider are divided into eight different bodies allowing 16 degrees of freedom.

Motorcycle modeling highlights two different dynamic modes:

- in-plane mode, which aims to describe longitudinal dynamics in straight running. It involves pitch, longitudinal speed and acceleration.
- out-of-plane mode, which aims to describe lateral dynamics in cornering situation. It involves roll, yaw, steering and lateral motion of the PTW.

The well-known Sharp'71 model presented in Sharp (1971) is used in this work to derive the lateral model of the scooter. The compromise between simplicity and ability to capture dynamics is the main motivation of this choice. It considers the motorcycle as a set of two rigid bodies joined at the steering axis with freedom, restrained by a linear steering damper. Compared to a single body model whose the input are the steering angle the Sharp'71 model takes into account the steering dynamics and considers the rider's torque applied on handlebar as the system input. It is obtained with a linearisation around straight-running with small-angle approximation assumption and also considering that the products between dynamic states are negligible. The obtained equations describe lateral dynamics and take into account coupling between longitudinal and lateral motions by considering the forward speed v_x as a time varying parameter. Tire relaxation is also considered by including the linear dynamics expression of F_{yf} and F_{yr} which are respectively, the front and rear lateral forces of tires. This dynamics is important to take into account because it plays an important role for the vehicle stability Sharp et al. (2004). As mentioned previously, in addition to tire dynamics, rider's torque τ, roll ϕ, yaw ψ, steering angle δ, lateral motion v_y and their dynamics define the whole of lateral dynamic state as:

Lateral motion:

$$m_{33}\dot{v}_y + m_{34}\ddot{\psi} + m_{35}\ddot{\phi} + m_{36}\ddot{\delta} = q_{34}v_x\dot{\psi} + Fy_f + Fy_r \tag{2}$$

Yaw motion:

$$\begin{aligned} m_{34}\dot{v}_y + m_{44}\ddot{\psi} + m_{45}\ddot{\phi} + m_{46}\ddot{\delta} = & \; q_{44}v_x\dot{\psi} + q_{45}v_x\dot{\phi} \\ & + q_{46}v_x\dot{\delta} + q_{47}Fy_f \\ & + q_{48}Fy_r \end{aligned} \tag{3}$$

Roll motion:

$$\begin{aligned} m_{35}\dot{v}_y + m_{45}\ddot{\psi} + m_{55}\ddot{\phi} + m_{56}\ddot{\delta} = & \; q_{51}\phi + q_{52}\delta + q_{54}v_x\dot{\psi} \\ & + q_{56}v_x\dot{\delta} \end{aligned} \tag{4}$$

Steering motion:

$$\begin{aligned} m_{36}\dot{v}_y + m_{46}\ddot{\psi} + m_{56}\ddot{\phi} + m_{66}\ddot{\delta} = & \; q_{52}\phi + q_{62}\delta + q_{64}v_x\dot{\psi} \\ & + q_{65}v_x\dot{\phi} + q_{66}\dot{\delta} \\ & + q_{67}Fy_f + \tau \end{aligned} \tag{5}$$

Tire's motion:

$$\begin{cases} \dot{F}y_f = q_{71}v_x\phi + q_{72}v_x\delta + q_{73}v_y + q_{74}\dot{\psi} \\ \qquad\quad + q_{76}\dot{\delta} + q_{77}v_xFy_f \\ \dot{F}y_r = q_{81}v_x\phi + q_{83}v_y + q_{84}\dot{\psi} + q_{76}\dot{\delta} \\ \qquad\quad + q_{88}v_xFy_f \end{cases} \tag{6}$$

Please refer to Appendix for the expressions of the coefficients m_{ij} and q_{ij}. Adding the two trivial expressions $\dot{\phi} = \dot{\phi}$ and $\dot{\delta} = \dot{\delta}$ we get 8 dynamic equations and the problem can be easily transformed under matrix formalism. Lateral model of the scooter can be expressed by the following Linear Parameter Varying (LPV) descriptor system:

$$M\dot{\tilde{x}} = QV(v_x)\tilde{x} + R\tau \tag{7}$$

where $\tilde{x} = [\phi, \delta, v_y, \dot{\psi}, \dot{\phi}, \dot{\delta}, F_{yf}, F_{yr}]^T$ denotes the vector of states. $V(v_x)$ is a parameter-varying matrix related to the forward velocity v_x, whereas $M = [m_{ij}]_{8*8}$, $Q = [q_{ij}]_{8*8}$ and R are time-invariant parameters. Equation (7) can be transformed into:

$$\dot{\tilde{x}} = \tilde{A}(v_x)\tilde{x} + \tilde{B}\tau \tag{8}$$

with $\tilde{A}(v_x) = M^{-1}QV(v_x)$ the state matrix and $\tilde{B} = M^{-1}R$ the input vector. To set the scooter parameters given in Appendix and compute the terms q_{ij} and m_{ij} of the model, a CAD model and static measurements were performed. Vehicle, rider with equipment and additional weight resulting from the top case including computer and external battery to power it are considered around 200 kg. Stability analysis shows that without controller the range of stability defining by a negative real part of each pole is included between 20 and 87 km/h where the wobble mode appears. This large stability range can be explained by the fact that the vehicle center of gravity is very low due the three heavy NiCad batteries installed under the footrest.

As discussed previously, rider's torque τ, considered as the primary input of the model, applied on the handlebar is very difficult to measure and our instrumentation architecture does not allow to get this measure that is why rider's action is considered as an Unknown Input (UI). After having studied possibilities and relevance of measured state combinations in order to ensure observability properties, it comes three pertinent measures: the steering angle δ given by the steering encoder (fig. 1), the roll rate $\dot{\phi}$ and the yaw rate $\dot{\psi}$ given by the IMU placed approximately on the center of gravity under the seat (fig. 1). Regarding the chosen measures and observability conditions, the equation (8) needs a transformation to get strong observability property. The roll angle is now considered as a second UI and the complete state-space representation for observer design can be expressed as follows:

$$\begin{cases} \dot{x} = A(v_x)x + D(v_x)d \\ y = Cx \end{cases} \tag{9}$$

with x the state vector without the roll angle ϕ, $y = \left[\, \delta \;\, \dot{\psi} \;\, \dot{\phi} \,\right]^T$ the vector of measures, $D(v_x) = \left[\, B \;\, \tilde{D}(v_x) \,\right]$ the UI matrix and $d = \left[\, \tau \;\, \phi \,\right]^T$ the UI vector. $\tilde{D}(v_x)$ is the corresponding extract vector from $\tilde{A}(v_x)$ according to the roll state. One can remark that the equation $dim(y) > dim(d)$ is verified.

4. OBSERVER DESIGN PROCEDURE

The present section is based on our recent previous work. For details, please refer to Damon et al. (2016). In this section the method to design the unknown input observer considering the LPV model of the scooter (9) is summarized. The approach considers Lyapunov theory associated with LMI tools to guarantee the asymptotically convergence toward zero of the state estimation errors.

Consider the following unknown input observer:

$$\begin{cases} \dot{z} = N(v_x, \dot{v}_x)z + L(v_x, \dot{v}_x)y \\ \hat{x} = z - H(v_x)y \end{cases} \tag{10}$$

Note that the matrices $N(v_x, \dot{v}_x)$, $L(v_x, \dot{v}_x)$ and $H(v_x)$ are parameter varying and not fixed a priori which offers a flexibility in the design as discussed in Ichalal and Mammar (2015) and Ichalal et al. (2015), their structures will be defined later. $\hat{x}$ is the estimate vector of state.

Writing the state estimation error as follows:

$$e = x - \hat{x} \tag{11}$$

its dynamics is expressed as follows:

$$\begin{aligned} \dot{e} = N(v_x, \dot{v}_x)e + [P(v_x)A(v_x) + \dot{P}(v_x, \dot{v}_x) \\ - L(v_x, \dot{v}_x)C - N(v_x, \dot{v}_x)P(v_x)]x + P(v_x)D(v_x)d \end{aligned} \tag{12}$$

with $P(v_x) = I + H(v_x)C$ and I the identity matrix with corresponding size.

In order to decouple the state x and the unknown input d from the state estimation error dynamics, the following three conditions should be satisfied:

- $N(v_x, \dot{v}_x)$ must be Hurwitz
- $P(v_x)A(v_x) + \dot{P}(v_x, \dot{v}_x) - L(v_x, \dot{v}_x)C$ $- N(v_x, \dot{v}_x)P(v_x) = 0$
- $P(v_x)D(v_x) = 0$

Note that the third condition admits a solution if and only if $rank(CD(v_x)) = rank(D(v_x))$ which is satisfied for $v_x \neq 0$.

Under these three conditions (12) becomes:

$$\dot{e} = [P(v_x)A(v_x) + \dot{P}(v_x, \dot{v}_x) - K(v_x, \dot{v}_x)C]e \tag{13}$$

with $K(v_x, \dot{v}_x) = L(v_x, \dot{v}_x) + N(v_x, \dot{v}_x)H(v_x)$

Then the well known sector nonlinear approach Tanaka and Wang (2001) is used to transform the problem into a polytopic form:

$$\begin{cases} P(v_x)A(v_x) + \dot{P}(v_x, \dot{v}_x) = \sum_{i=1}^{r} h_i\left(v_x, \dot{v}_x\right) \mathcal{A}_i \\ K(v_x, \dot{v}_x) = \sum_{i=1}^{r} h_i\left(v_x, \dot{v}_x\right) K_i \end{cases} \tag{14}$$

With $r = 4$ the number of sub-models coming directly from the two nonlinearities v_x and $\dot{v}_x$. $h_i(.)$ are the membership functions satisfying the convex sum property as explained in Damon et al. (2016).

Let us consider the following quadratic Lyapunov function to address the stability problem:

$$V(e) = e^T X e, \quad X = X^T > 0 \tag{15}$$

whose time derivatives $\dot{V}(e)$ leads to:

$$\begin{aligned} \dot{V}(e) = e^T \sum_{i=1}^{r} h_i\left(v_x, \dot{v}_x\right)(\mathcal{A}_i^T X + X\mathcal{A}_i \\ -XK_iC - C^T K_i^T X)e \end{aligned} \tag{16}$$

Let us introduce the change of variables $\bar{K}_i = XK_i,\ i = 1, ..., 4$. Since the weighting functions satisfy the convex sum property, sufficient conditions ensuring $\dot{V}(e(t)) < 0$ are given by the following LMIs:

$$\mathcal{A}_i^T X + X\mathcal{A}_i - \bar{K}_iC - C^T\bar{K}_i^T < 0, i = 1, ..., r \tag{17}$$

with $X = X^T > 0$.

Find gain matrices $\bar{K}_i$ and matrix X satisfying (17) ensure asymptotic convergence to zero of the state estimation error. Since $K(v_x, \dot{v}_x)$ is defined by expression (14) it can be reconstructed from $K_i = X^{-1}\bar{K}_i, \quad i = 1, ..., r$. Then the observer can be completely defined by computing $N(v_x, \dot{v}_x)$, $L(v_x, \dot{v}_x)$ and $H(v_x)$ (see Damon et al. (2016) for more details).

The UIO allows to estimate the whole of state vector x but does not give information about UI, that is why we need to proceed into a reconstruction of the UI based on estimated states and output derivatives. To estimate the state and output time derivatives, a High-Order sliding mode differentiator is used. For more detail on this type of signal differentiation algorithm, please refer to Levant (2003). It provides an estimation of the steering rate $\hat{\dot{\delta}}$, steering acceleration $\hat{\ddot{\delta}}$, yaw acceleration $\hat{\ddot{\psi}}$, roll acceleration $\hat{\ddot{\phi}}$ from the measured states and the lateral speed $\hat{\dot{v}}_y$ form the estimated states.

The roll angle reconstruction is based on the roll motion equation (4) which leads to:

$$\hat{\phi} = f_1(\hat{\dot{v}}_y, \hat{\ddot{\psi}}, \hat{\ddot{\phi}}, \hat{\ddot{\delta}}, \delta, \dot{\psi}, \hat{\dot{\delta}}, v_x) \tag{18}$$

And the rider's torque reconstruction is based on the roll steering equation (5) which leads to:

$$\hat{\tau} = f_2(\hat{\dot{v}}_y, \hat{\ddot{\psi}}, \hat{\ddot{\phi}}, \hat{\ddot{\delta}}, \hat{\phi}, \delta, \dot{\psi}, \dot{\phi}, \hat{\dot{\delta}}, \hat{Fy}_f, v_x)$$

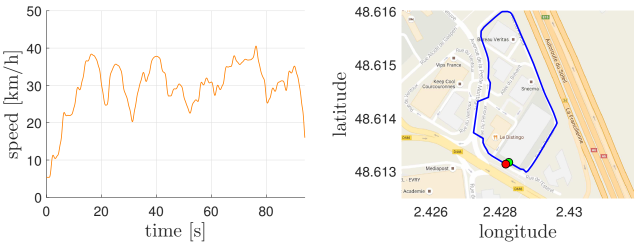
5. EXPERIMENTAL RESULTS

This section aims to present experimental results to validate the proposed Unknown Input Observer for lateral and steering dynamics estimation.

Let us remind that the observer is only based on te motorcycle model and are independent of the controller which is the driver in real case. Even under this assumption the UIO is able to estimate every lateral dynamic states and reconstruct the rider's torque.

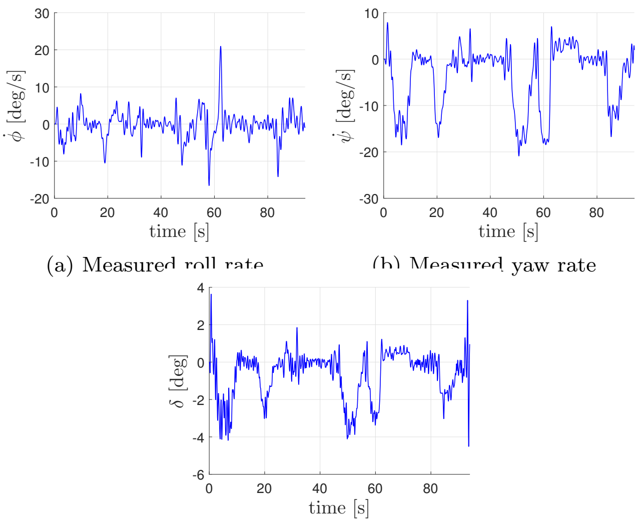
A scenario realized on urban scenic road was performed with normal riding behavior and good environmental conditions. As said in previous section our instrumentation architecture does not allow to measure the rider's torque, lateral tire forces, lateral speed and steering rate that is why several figures below show only estimation for concerned states. Roll angle and lateral acceleration given by the IMU are the only two dynamic states which can validate the roll and acceleration estimation given by the observer.

Fig. 2 shows the riding scenario and is composed of straight lines, narrow and big turns. One can remark that there is no restriction about forward speed expect $v_x = 0$.



(a) Longitudinal speed (b) Vehicle trajectory

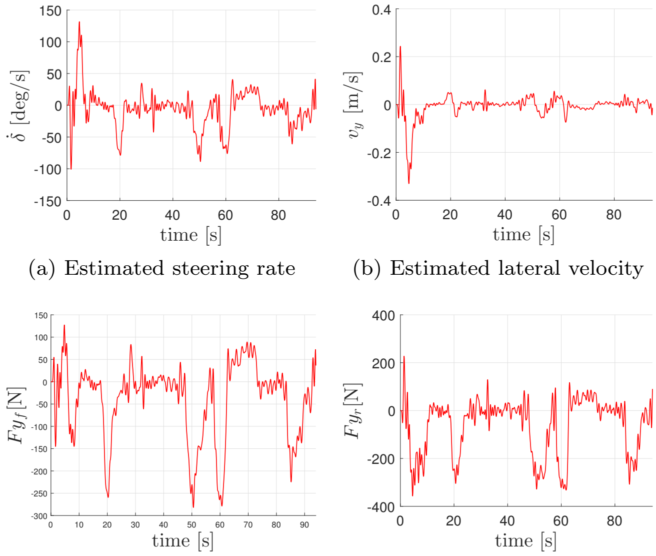
Fig. 2. Riding scenario



(a) Measured roll rate (b) Measured yaw rate

(c) Measured steering angle

Fig. 3. Measured states



(a) Estimated steering rate (b) Estimated lateral velocity

(c) Estimated front lateral tire force (d) Estimated rear lateral tire force

Fig. 4. State's estimations

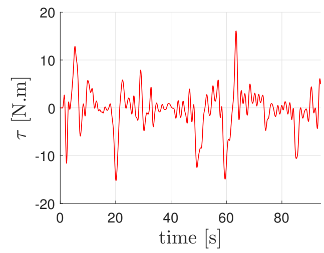
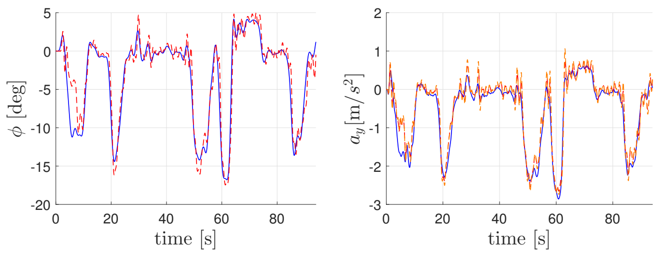


Fig. 5. Estimated rider's torque



(a) Measured (blue) and estimated (red) roll angle (b) Measured (blue) and estimated (orange,red) lateral acceleration

Fig. 6. Validated dynamic states

The measures needed by the observer are given in fig. 2. They come from the encoder for the steering angle and from the IMU for the yaw and roll rate.

Fig. 4 shows the estimation of the lateral dynamic states whereas fig. 5 presents the results of the estimated rider's torque.

Lateral acceleration: blue measure, red estimated with sum force $a_y = (Fy_f + Fy_r)/M$ and orange estimated with lateral motion $a_y = \dot{v}_y + \dot{\psi} v_x$

This last figures demonstrate the ability of the UIO observer to perfectly estimate the lateral dynamic states and to reconstruct the roll angle and the rider's torque on a real driving scenario realized with normal riding behavior.

6. CONCLUSION

In this paper we have discussed the necessity to validate observers with real instrumented vehicle. This article deals with off-line validation of the proposed UIO in Damon et al. (2016) to estimate lateral motorcycle dynamic states and to reconstruct unknown inputs during real riding scenarios. A light scooter equipped with sensors are introduced and an acquisition architecture is proposed. The motorcycle model used for observer design is derived from the well-known two-bodies Sharp's model. A CAD model of the scooter combined with geometric measurements provide needed parameters. Then, a test endorse the theoretical results by showing the performances of the UIO in a real scenario. Next step is to perform on-line estimation and to use the estimated variables for detecting critical situations such as a dangerous roll angle for example.

7. APPENDIX

Variables, matrices and notations	
v_x, v_y	longitudinal and lateral speeds
ϕ, ψ, δ	roll, yaw and steer angles
τ	rider torque
F_{y_f}, F_{y_r}	lateral forces
$\dot{x}$, $\ddot{x}$	time derivatives of the var x
$\hat{x}$	estimate of a variable x
x^T	transpose of vector or matrix x
x_f, x_r	denotes front and rear
$\tilde{A}(v_x)$, $A(v_x)$	state matrices
$\tilde{B}$, B	input vectors
$\tilde{D}(v_x)$, $D(v_x)$	unknown input matrix
C	observation matrices

Motorcycle parameters		
g	[9.81] m/s	gravity
ϵ	[0.4014] rad	caster angle
η	[0.0783] m	mechanical trail
K	[15] N.m/(rad/s)	steering damper
Z_f	[-854.8] N	front vertical load
C_{rxz}	[1.6] kg/m	rear frame inertia product
M_f, M_r	[15,190] kg	body mass
j, h	[0.41,0.5] m	geometric dimensions (*)
k, e	[0.75,0.0051] m	geometric dimensions (*)
l_f, l_r	[0.82,0.48] m	geometric dimensions (*)
R_f, R_r	[0.207,0.215] m	wheel radius
i_{fy}, i_{ry}	[0.40,0.48] kg/m	wheel inertia around Y
I_{fx}, I_{rx}	[2.5,21] kg/m	body inertia around X
I_{fz}, I_{rz}	[0.2,9] kg/m	body inertia around Z
C_{f1}, C_{r1}	[12571,13040] N/rad	tire cornering stiffness
C_{f2}, C_{r2}	[483.8,374.4] N/rad	tire camber stiffness
σ_f, σ_r	[0.25,0.25] m	tire relaxation coefficient

(*) For more details please refer to Sharp (1971).

Matrices terms m_{ij} and q_{ij} $\quad i, j = 1..8$

$m_{33} = M_f + M_r$, $m_{34} = M_f k$, $m_{35} = M_f j + M_r h$, $m_{36} = M_f e$, $m_{44} = M_f k^2 + I_{rz} + I_{fx} sin^2(\epsilon) + I_{fz} cos^2(\epsilon)$, $m_{45} = M_f jk - C_{rxz} + (I_{fz} - I_{fx}) sin(\epsilon) cos(\epsilon)$, $m_{46} = M_f ek + I_{fz} cos(\epsilon)$, $m_{55} = M_f j^2 + M_r h^2 + I_{rx} + I_{fx} cos^2(\epsilon) + I_{fz} sin^2(\epsilon)$, $m_{56} = M_f ej + I_{fz} sin(\epsilon)$, $m_{66} = I_{fz} + M_f e^2$

$q_{34} = -M_f - M_r$, $q_{44} = -M_f k$, $q_{45} = i_{fy}/R_f + i_{ry}/R_r$, $q_{46} = sin(\epsilon) i_{fy}/R_f$, $q_{47} = l_f$, $q_{48} = -l_r$, $q_{51} = (M_f j + M_r h)g$, $q_{52} = M_f eg - \eta Z_f$, $q_{54} = -M_f j - M_r h - i_{fy}/R_f - i_{ry}/R_r$, $q_{56} = -cos(\epsilon) i_{fy}/R_f$, $q_{62} = (M_f eg - \eta Z_f) sin(\epsilon)$, $q_{64} = -M_f e - sin(\epsilon) i_{fy}/R_f$, $q_{65} = cos(\epsilon) i_{fy}/R_f$, $q_{66} = -K$, $q_{67} = -\eta$, $q_{71} = C_{f2}/\sigma_f$, $q_{72} = (C_{f2} sin(\epsilon) + C_{f1} cos(\epsilon))/\sigma_f$, $q_{73} = -C_{f1}/\sigma_f$, $q_{74} = -l_f C_{f1}/\sigma_f$, $q_{76} = \eta C_{f1}/\sigma_f$, $q_{77} = -1/\sigma_f$, $q_{81} = C_{r2}/\sigma_r$, $q_{83} = -C_{r1}/\sigma_r$, $q_{84} = l_r C_{r1}/\sigma_r$, $q_{88} = -1/\sigma_r$

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