Interval observer for LPV systems: Application to vehicle lateral dynamics

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Abstract: This paper presents a new method for guaranteed and robust estimation of sideslip angle and lateral tire forces with consideration of cornering stiffness variations resulting from changes in tire/road and driving conditions. An interval LPV observer with both measurable and unmeasurable time-varying parameters is proposed. The longitudinal velocity is treated as the online measured time-varying parameter and the cornering stiffness at front and rear tires are assumed to be unknown but bounded with a priori known bounds. The obtained results are no more punctual values but a set of acceptable values. The simulation is based on experimental data in order to prove the effectiveness of the proposed observers.

Keywords: Interval observer, uncertain systems, robustness, positive systems, eigenvalue assignment, state estimation, vehicle lateral dynamics.

1. INTRODUCTION

Accurate knowledge of state variables such as sideslip angle and tire forces is essential to improve the safety, handling, performance and comfort of vehicles. However, the complexity of the technical implementation and cost prohibitive for the installation of sensors to measure these important data make their integration into standard vehicles an unfeasible solution. Therefore, these variables must be estimated using observers and measurements from standard sensors such as gyro, accelerometer, etc.

In the literature, several studies have addressed the design of classic observers to estimate the vehicle lateral dynamic states using different approaches. For example, Luenberger observer, Kalman Filter (Venhovens and Naab (1999)), Extended Kalman filter (Satria and Best (2005)), Unknown input proportional-integral observer (Mammar et al. (2006)) and sliding mode observer (Stéphant et al. (2007)). Most of these studies have been based on the assumption that the cornering stiffness parameters are constant. This assumption is verified only when the vehicle is operating in the linear region of lateral forces (Fig. 2) and the road conditions are nominal. However, when the road friction changes, the nonlinear region is generally reached. Consequently, the vehicle approaches to its operational limit conditions and its response to the driver's inputs becomes less responsive making these parameters as an obstacle in developing a high performance estimator.

In (M'sirdi et al. (2005)), a sliding mode observer (SMO) has been used to identify the tire/road parameters. It is one of the popular robust approaches, in fact, the sliding surface ensures the robustness of the parameter variations. However, the main disadvantage of the sliding mode technique is the undesirable chattering phenomenon (Utkin et al. (2009)). Another popular approach is presented in (Hiraoka et al. (2004)) using adaptive observer. This method suffers from a significant disadvantage that is the existence of solution satisfying the sufficient conditions is not always guaranteed. Furthermore, an accurate estimate of the parameters requires that the system inputs to satisfy the conditions of persistent excitation. In (Ray (1997)), a extended Kalman-Bucy filtering (EKBF) is used to estimate lateral forces, which are treated as random variables. This method allows to achieve precise parameter estimates, but requires accurate knowledge of the model and noise statistics.

In the last decades, the development of the interval observer (Gouzé et al. (2000), Rapaport and Harmand (2002)) represent an alternative technique for robust estimation in the presence of parameter uncertainties, unknown inputs or measurement disturbances. It becomes, a popular successful robust approach especially in biotechnological domain (Rapaport and Dochain (2005), Meslem et al. (2008)). Note that, interval observers can be defined as a pair of estimators based on Luenberger structure which provide a guaranteed bounds covering all admissible trajectories of system, using a priori known bounds on uncertain parameters and/or exogenous disturbances. The synthesis of these observers often uses an additional assumptions to prove the stability of the estimated bounds, the monotony and cooperativity (Smith (2008)). These properties keep the partial order between lower and upper trajectories.

Several interval observers are proposed in the literature. For instance, in Rami et al. (2008), Bolajraf et al. (2010) and Rami et al. (2013), interval observers for linear uncertain systems are presented. The necessary and sufficient conditions have been formulated in terms of linear programming. The case of the LPV and nonlinear systems

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are treated in Raïssi et al. (2010), Efimov et al. (2012) and Efimov et al. (2013) using the Lyapunov theory and linear matrix inequalities (LMIs). In this paper, an interval observer for LPV systems which contain unmeasured and measured uncertain parameters is proposed. The observer is based on a robust pole assignment depending on the parameter variation.

The present paper is organized as follows. Some preliminaries are given in section 2. In section 3, we present the uncertain LPV system of the vehicle lateral dynamics. The section 4 is devoted to the main result. The Experimental results are provided in section 5. A conclusion is drawn in section 6 and end the technical note.

2. PRELIMINARIES

The objective of this section is to provide some notations and basic definitions that are used throughout the paper.

- A vector with null components is denoted by $\mathbf{0}$.
- The absolute value of x is denoted by $|x|$.
- The norm L_∞ of x is denoted by $\|x\|$.
- The left and right endpoints of an interval $[x]$ (resp. a matrix M) are denoted respectively by x^- and x^+ (resp. M^- and M^+) such as $[x] = [x^-, x^+]$ (resp. $[M] = [M^-, M^+]$).
- All the inequalities must be interpreted element wise.
- Let a vector $x \in \mathbb{R}^n$ or a matrix $A \in \mathbb{R}^{n\times n}$, one denotes $\overline{x} = \max\{0, x\}$, $\underline{x} = \overline{x} - x$ or $\overline{A} = \max\{0, A\}$, $\underline{A} = \overline{A} - A$.
- The eigenvalues of a matrix A are denoted λ.
- A real matrix A is called Hurwitz if all its eigenvalues have strictly negative real part ($Re(\lambda < 0)$).
- A real matrix A is called Metzler if all its elements outside the main diagonal are positive ($a_{ij} \geq 0,\ \ \forall i \neq j$).
- A continuous-time linear system is cooperative if its state matrix A is a Metzler matrix.

Lemma 1 (Gouzé et al. (2000)) For a Metzler matrix A, the cooperative system:

$$\dot{x}(t) = Ax(t) + d(t)$$

with $x \in \mathbb{R}^n$ and $d : \mathbb{R} \to \mathbb{R}^n_+$ is said to be positive if $x(0) \geq 0$ then $x(t) \geq 0$, $\forall t \geq 0$.
Lemma 2 (Efimov et al. (2012)) Let $x \in [x^-, x^+]$ be a variable vector, then for a variable matrix $\Delta A \in \mathbb{R}^{n\times n}$ such as $\Delta A^- \leq \Delta A \leq \Delta A^+$ for some $\Delta A^-, \Delta A^+ \in \mathbb{R}^{n\times n}$, then

$$\begin{aligned} \underline{\Delta A}^+\underline{x}^+ - \overline{\Delta A}^+\underline{x}^- - \underline{\Delta A}^-\overline{x}^+ + \overline{\Delta A}^-\overline{x}^- \leq \Delta A x \leq \\ \overline{\Delta A}^+\overline{x}^+ - \underline{\Delta A}^+\overline{x}^- - \overline{\Delta A}^-\underline{x}^+ + \underline{\Delta A}^-\underline{x}^- \end{aligned} \quad (1)$$

3. VEHICLE LATERAL MODEL

Vehicle lateral dynamics could be modeled by a bicycle model which a two degree of freedom (2-DOF) vehicle model with sideslip angle and yaw rate as the states. The dynamics equations can be represented by (Rajamani (2011)):

$$\begin{cases} mv_x(\dot{\beta} + r) = F_{yf} + F_{yr} \\ I_z\dot{r} = l_f F_{yf} - l_r F_{yr} \end{cases} \quad (2)$$

where m, I_z, l_r, l_f denote respectively the mass of the vehicle, the yaw moment and the distances from the rear

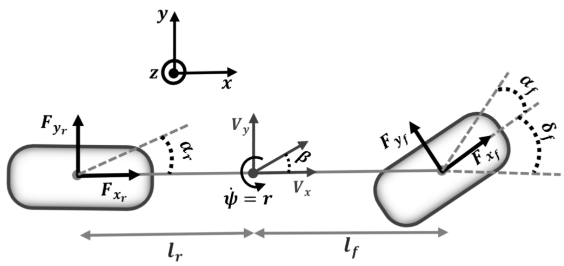

Fig. 1. Bicycle Model.

and the front axle to the center of gravity. v_x is a time-varying longitudinal velocity, β is the sideslip angle of the vehicle and r is the yaw rate. F_{yr} and F_{yf} are the lateral rear and front forces respectively.

The nonlinear forces F_{yf} and F_{yr} are usually functions of the wheel sideslip angle and wheel longitudinal slip (Dugoff et al. (1970), Pacejka and Bakker (1991), Burckhardt (1993), Kiencke and Nielsen (2000)). Using Pacejka's magic formula (Pacejka and Bakker (1991)), the lateral forces are given by:

$$F_{yi} = D_i sin(C_i tan^{-1}(B_i(1 - E_i)\alpha_i + E_i tan^{-1}(B_i\alpha_i))) \quad (3)$$

where $i = \{r, f\}$ denotes rear and front of the vehicle. D_i, C_i, B_i and E_i are the characteristic constants of the tires. α_f and α_r are respectively the front and rear sideslip angles of the tires expressed by (Cheng et al. (2011)):

$$\begin{cases} \alpha_f = \delta_f - \beta - tan^{-1}(\dfrac{lf}{v_x} r cos(\beta)) \\ \alpha_r = -\beta + tan^{-1}(\dfrac{lf}{v_x} r cos(\beta)) \end{cases} \quad (4)$$

For small variations of the sideslip angle ($\leq 8°$), (4) may be simplified as follows:

$$\begin{cases} \alpha_f = \delta_f - \beta - \dfrac{l_f}{v_x} r \\ \alpha_r = -\beta + \dfrac{l_r}{v_x} r \end{cases} \quad (5)$$

F_{yf} and F_{yr} are nonlinear forces but in this work the forces are considered linear with respect to the sideslip angles of the tires (linear approximation of (3)):

$$\begin{cases} F_{yf} = c_f\alpha_f \\ F_{yr} = c_r\alpha_r \end{cases} \quad (6)$$

c_f and c_r denote respectively the cornering stiffness of front and rear tires and they correspond to the slope at the origin (Fig. 2). These parameters are closely related to road friction. If road friction changes or if the nonlinear tire region is reached, cornering stiffness varies. Consequently, we consider in this study that the cornering stiffness in (6) are expressed as a linear part (denoted c_{i0}) and an uncertainty term (denoted Δc_i) assumed to be unknown but bounded with a priori known bounds (Fig. 2):

$$\begin{cases} F_{yf} = (c_{f_0} + \Delta c_f)\alpha_f \\ F_{yr} = (c_{r_0} + \Delta c_r)\alpha_r \end{cases} \quad (7)$$

Gathering equations (2), (5) and (6) leads to the following model:

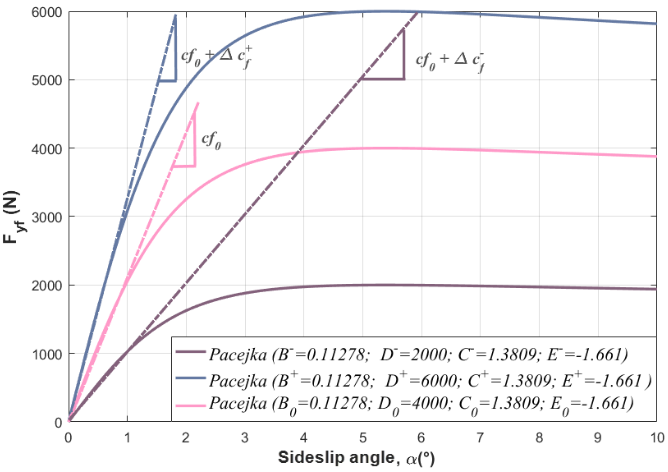

Fig. 2. Pacejka lateral force model characteristics.

$$\begin{cases} \dot{x}(t) = (A_0(\rho(t)) + \Delta A(\xi(t)))x(t) + B(\rho(t), \xi(t))u(t) \\ y(t) = Cx(t) + e(t) \end{cases} \tag{8}$$

The state vector $x(t)$ comprises slideslip angle and yaw rate $x(t) = [\beta \quad r]^T$. $y(t)$ is the measurable output with an observation matrix $C = [0 \quad 1]$ and a measurement noise $e(t)$. The input of the system is the steering angle δ_f. $\rho(t)$ and $\xi(t)$ represent respectively the measurable and unmeasurable scheduling parameters, where: $\rho(t) = \begin{bmatrix} \frac{1}{v_x} & \frac{1}{v_x^2} \end{bmatrix}^T$ and $\xi(t) = [\Delta c_f \quad \Delta c_r]^T \in \Xi$ is the vector of uncertain parameters with a known interval Ξ given by:

$$\Xi = \begin{bmatrix} [\Delta c_f^-, \Delta c_f^+] \\ [\Delta c_r^-, \Delta c_r^+] \end{bmatrix} \tag{9}$$

For simplicity of the notations, we adopt M_ρ and $M_{\rho,\xi}$ as a shorthand of $M(\rho(t))$ and $M(\rho(t), \xi(t))$ respectively.

The state space matrices $A_{0\rho}$, $\Delta A_{\rho,\xi}$ and $B(\xi(t), \rho(t))$ are defined by:

$$A_{0\rho} = \begin{bmatrix} -\dfrac{c_{f_0} + c_{r_0}}{m}\rho_1(t) & \dfrac{(c_{r_0} l_r - c_{f_0} l_f)}{m}\rho_2(t) - 1 \\ \dfrac{(c_{r_0} l_r - c_{f_0} l_f)}{I_z} & -\dfrac{c_{r_0} {l_r}^2 + c_{f_0} {l_f}^2}{I_z}\rho_1(t) \end{bmatrix} \tag{10}$$

$$\Delta A_{\rho,\xi} = \begin{bmatrix} -\dfrac{\Delta c_f + \Delta c_r}{m}\rho_1(t) & \dfrac{(\Delta c_r l_r - \Delta c_f l_f)}{m}\rho_2(t) \\ \dfrac{(\Delta c_r l_r - \Delta c_f l_f)}{I_z} & -\dfrac{\Delta c_r {l_r}^2 + \Delta c_f {l_f}^2}{I_z}\rho_1(t) \end{bmatrix} \tag{11}$$

$$B(\xi(t), \rho(t)) = \begin{bmatrix} \dfrac{c_{f0} + \Delta c_f}{m}\rho_1(t) \\ \dfrac{(c_{f0} + \Delta c_f) l_f}{I_z} \end{bmatrix} \tag{12}$$

where $\rho_1(t)$ and $\rho_2(t)$ are the components of the time-varying parameter vector $\rho(t)$ (i.e. $\rho_1(t) = \frac{1}{v_x}$, $\rho_2(t) = \frac{1}{v_x^2}$).

4. INTERVAL OBSERVER DESIGN FOR LATERAL DYNAMICS ESTIMATION

In this section, an interval observer is presented to estimate the sideslip angle and lateral tire forces using Pacejka's model. A block diagram of the estimation procedure is illustrated in figure 3. It includes:

(1) An interval observer which uses the measured variables, longitudinal velocity, yaw rate and steering angle to obtain the upper and lower bounds of sideslip angle and yaw rate.
(2) An algebraic estimator based on Pacejka's equations to obtain the lateral forces bounds.

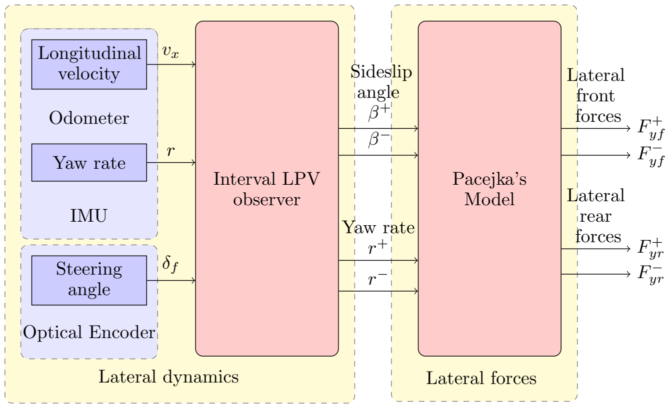

Fig. 3. Schematic overview of estimation methodology.

The construction of an observer interval for (8) requires the following assumptions:

Assumption 1. There exist constants $\mathcal{X} \geq 0$ and $\mathcal{U} \geq 0$ such that $\|x\| \leq \mathcal{X}$, $\|u\| \leq \mathcal{U}$.
Assumption 2. The pair $(A_{0\rho}, C)$ is detectable $\forall \rho(t)$, $t \geq 0$.
Assumption 3. There exist u^-, u^+, e^-, e^+, and matrices ΔA_ρ^-, ΔA_ρ^+, B_ρ^-, B_ρ^+ such that:

$$u^- \leq u(t) \leq u^+ , \quad e^- \leq e(t) \leq e^+ \ \mid \ \mathbf{0} \in [e^-, e^+] \tag{13}$$

$$\Delta A_\rho^- \leq \Delta A_{\rho,\xi} \leq \Delta A_\rho^+ , \quad B_\rho^- \leq B_{\rho,\xi} \leq B_\rho^+ \tag{14}$$

The matrices ΔA_ρ^-, ΔA_ρ^+, B_ρ^-, B_ρ^+ can be computed under the assumption that the unmeasured parameter $\xi(t)$ satisfies (7):

$$\Delta A_\rho^- = \begin{bmatrix} -\dfrac{\Delta c_f^+ + \Delta c_r^+}{m}\rho_1(t) & \dfrac{\left(\Delta c_r^- l_r - \Delta c_f^+ l_f\right)}{m}\rho_2(t) \\ \dfrac{\left(\Delta c_r^- l_r - \Delta c_f^+ l_f\right)}{I_z} & -\dfrac{\Delta c_r^+ {l_r}^2 + \Delta c_f^+ {l_f}^2}{I_z}\rho_1(t) \end{bmatrix} \tag{15}$$

$$\Delta A_\rho^+ = \begin{bmatrix} -\dfrac{\Delta c_f^- + \Delta c_r^-}{m}\rho_1(t) & \dfrac{\left(\Delta c_r^+ l_r - \Delta c_f^- l_f\right)}{m}\rho_2(t) \\ \dfrac{\left(\Delta c_r^+ l_r - \Delta c_f^- l_f\right)}{I_z} & -\dfrac{\Delta c_r^- {l_r}^2 + \Delta c_f^- {l_f}^2}{I_z}\rho_1(t) \end{bmatrix} \tag{16}$$

$$B_\rho^- = \begin{bmatrix} \dfrac{c_{f0} + \Delta c_f^-}{m}\rho_1(t) \\ \dfrac{(c_{f0} + \Delta c_f^-) l_f}{I_z} \end{bmatrix}, \quad B_\rho^+ = \begin{bmatrix} \dfrac{c_{f0} + \Delta c_f^+}{m}\rho_1(t) \\ \dfrac{(c_{f0} + \Delta c_f^+) l_f}{I_z} \end{bmatrix} \tag{17}$$

4.1 Interval observer structure

Under assumptions 1,2, and 3 and according to lemma 1, the following proposed system:

$$\begin{cases} \dot{x}^+(t) = (A_{0\rho} - L_\rho C)x^+ + (\overline{\Delta A}_\rho^+ \overline{x}^+ - \underline{\Delta A}_\rho^+ \overline{x}^- - \overline{\Delta A}_\rho^- \underline{x}^+ \\ +\underline{\Delta A}_\rho^- \underline{x}^-) + B_\rho^+ u^+ + L_\rho y + |L_\rho| e^+ \\ \\ \dot{x}^-(t) = (A_{0\rho} - L_\rho C)x^- + (\underline{\Delta A}_\rho^+ \underline{x}^+ - \overline{\Delta A}_\rho^+ \underline{x}^- - \underline{\Delta A}_\rho^- \overline{x}^+ \\ +\overline{\Delta A}_\rho^- \overline{x}^-) + B_\rho^- u^- + L_\rho y + |L_\rho| e^- \\ \\ x^-(t_0) \le x(t_0) \le x^+(t_0) \end{cases} \tag{18}$$

is a LPV interval observer for the system (8) if:

$$x^-(t) \le x(t) \le x^+(t), \quad \forall t \ge t_0 \tag{19}$$

The inequality (19) is satisfied if the upper and lower estimation errors $e^-(t) = x(t) - x^-(t)$ and $e^+(t) = x^+(t) - x(t)$ are defined positive for all initial conditions $e^-(t_0) = x(t_0) - x^-(t_0) \ge 0$ and $e^+(t_0) = x^+(t_0) - x(t_0) \ge 0$.

Dynamics of interval estimation errors are given by:

$$\begin{cases} \dot{e}^+(t) = (A_{0\rho} - L_\rho C)e^+(t) + d^+(t) \\ \dot{e}^-(t) = (A_{0\rho} - L_\rho C)e^-(t) + d^-(t) \end{cases} \tag{20}$$

with

$$\begin{cases} d^+(t) = (\overline{\Delta A}_\rho^+ \overline{x}^+ - \underline{\Delta A}_\rho^+ \overline{x}^- - \overline{\Delta A}_\rho^- \underline{x}^+ + \underline{\Delta A}_\rho^- \underline{x}^-) - \\ \Delta A_{\rho,\xi} + B_\rho^+ u^+ - B_{\rho,\xi} u + |L_\rho| e^+ - L_\rho e \\ d^-(t) = \Delta A_{\rho,\xi} - (\underline{\Delta A}_\rho^+ \underline{x}^+ - \overline{\Delta A}_\rho^+ \underline{x}^- - \underline{\Delta A}_\rho^- \overline{x}^+ + \\ \overline{\Delta A}_\rho^- \overline{x}^-) + B_{\rho,\xi} u - B_\rho^- u^- + L_\rho e - |L_\rho| e^- \end{cases} \tag{21}$$

The input $d^+(t)$ and $d^-(t)$ are nonnegative for all $t \ge t_0$ due to lemma 2 and assumption 3. Using this result and the fact that $A_{0\rho} - L_\rho C$ is Metzler by construction, then according to lemma 1, the estimation errors are positive for all $e^-(t_0) \ge 0$ and $e^+(t_0) \ge 0$.

4.2 Eigenvalue Assignment Problem

The objective of this section is to calculate the gain $L_\rho = [l_{1\rho}, l_{2\rho}]^T$ such that the matrix $(A_{0\rho} - L_\rho C)$ is Metzler and Hurwitz $\forall \rho(t)$. If the gain L_ρ is chosen such that $(A_{0\rho} - L_\rho C)$ is Metzler then the proposed observer is covering all possible state trajectories of (8). Furthermore, the gain L_ρ must ensure the stability and convergence of the interval observer. These two constraints are hard constraints, meaning that in some cases they can not be satisfied simultaneously (See discussion in remark 2). However, the vehicle model presented in section 3 satisfies the above constraints and a gain L_ρ can be calculated to ensure stability and cooperativity of the matrix $(A_{0\rho} - L_\rho C)$.

To study the convergence of the observer, we consider the total error given by:

$$\begin{aligned} e(t) &= e^+(t) - e^-(t) \\ &= x^+(t) - x^-(t) \end{aligned} \tag{22}$$

The dynamic of the total observation error $e(t)$ can be expressed as follows:

$$\begin{aligned} \dot{e}(t) &= \dot{e}^+(t) - \dot{e}^-(t) \\ &= (A_{0\rho} - L_\rho C)e(t) + \delta d(t) \end{aligned} \tag{23}$$

where

$$A_{0\rho} - L_\rho C =$$

$$\begin{bmatrix} -\dfrac{c_{f_0} + c_{r_0}}{m}\rho_1(t) & -l_{1\rho} + \dfrac{c_{r_0} l_r - c_{f_0} l_f}{m}\rho_2(t) - 1 \\ \dfrac{c_{r_0} l_r - c_{f_0} l_f}{I_z} & -l_{2\rho} - \dfrac{c_{f_0} l_f^2 + c_{r_0} l_r^2}{I_z}\rho_1(t) \end{bmatrix} \tag{24}$$

and

$$\delta d(t) = d^+(t) - d^-(t)$$

Stability of total error is ensured when the eigenvalues of $A_{0\rho} - L_\rho C$ have a strictly negative real parts $\forall \rho(t)$. Moreover, to ensure positivity, all the $A_{0\rho} - L_\rho C$ elements outside the main diagonal must be nonnegative.

Remark 1. One can notice that the given vehicle is understeering, thus $c_{r_0} l_r - c_{f_0} l_f > 0$.

Due to remark 1 and the fact that $\rho(t) > 0$, $\forall t \ge 0$ (By definition), an appropriate choice of the gain $l_{1\rho}$ to ensure the Metzler condition is:

$$l_{1\rho} = \tfrac{c_{r_0} l_r - c_{f_0} l_f}{m}\rho_2(t) - 1$$

Then, the eigenvalues of (23) becomes:

$$\begin{cases} \lambda_{1\rho} = -\dfrac{c_{f_0} + c_{r_0}}{m}\rho_1(t) \\ \lambda_{2\rho} = -l_{2\rho} - \dfrac{c_{f_0} l_f^2 + c_{r_0} l_r^2}{I_z}\rho_1(t) \end{cases} \tag{25}$$

Knowing that all parameters c_{f_0}, c_{r_0}, l_f, l_r, I_z and m are positive, it is clear that the eigenvalues of $A_{0\rho} - L_\rho C$ for all $\rho(t) > 0$, $t \ge 0$ are negative if the second component of the gain vector is defined positive $\forall t \ge 0$.

As a conclusion, the observer gain L_ρ which ensures the stability by placing the poles at (25) and ensures cooperativity of interval observer are given by:

$$\begin{pmatrix} l_{1\rho} \\ l_{2\rho} \end{pmatrix} \in \left\{ \left(\begin{array}{c} \dfrac{c_{r_0} l_r - c_{f_0} l_f}{m}\rho_2(t) - 1 \\ \tilde{a} \end{array} \right) \middle| , \tilde{a} \in [0, \infty) \right\} \tag{26}$$

Remark 2. In the present work, the synthesis of the proposed observer requires that the gain L_ρ ensures simultaneously the stability and cooperativity of the observation error. Note that this assumption is very conservative and generally difficult to satisfy. Therefore, in the case where we are not able to compute a gain $L(\rho(t))$ such that $A_{0\rho} - L_\rho C$ is Hurwitz and Metzler, we can find a time-varying non-singular matrix P_ρ such that the state matrix in the new base $z = P_\rho x$ is Hurwitz and Metzler matrix $\forall \rho(t)$ (Efimov et al. (2013)). The gain L_ρ can be computed for ensuring for example the asymptotic stability using a pole placement depending on the time-varying parameter $\rho(t)$ or using the LMI conditions. Thereafter, the time-varying change of coordinates is used to ensures the cooperativity (Metzler condition) of the observation error. However, there are still some problems to be overcome, essentially, the problem of practical implementation because the solution based on the time varying change of coordinates requires online resolution of a differential matrix equation.

4.3 Algebraic estimation of tire forces bounds

The idea now is to estimate the lateral forces using the algebraic formula of the linearized Pacejka's model and the bounds previously estimated, we can express the upper and lower bounds of lateral forces by:

$$\begin{cases} F_{yf}^{+} = c_f^{+}\alpha_f^{+} \\ F_{yf}^{-} = c_f^{-}\alpha_f^{-} \\ F_{yr}^{+} = c_r^{+}\alpha_r^{+} \\ F_{yr}^{-} = c_r^{-}\alpha_r^{-} \end{cases} \tag{27}$$

The measured parameter $\rho(t)$, the steering angle δ_f, the upper and lower bounds ψ^{+}, ψ^{-}, β^{+} and β^{-} of yaw rate and sideslip angle are then used for compute the bounds of tire slip angles α_f and α_r, where:

$$\begin{cases} \alpha_f^{-} = \delta_f - \beta^{+} - l_f\rho_1(t)r^{+} \\ \alpha_f^{+} = \delta_f - \beta^{-} - l_f\rho_1(t)r^{-} \\ \alpha_r^{-} = -\beta^{+} + l_r\rho_1(t)r^{-} \\ \alpha_r^{+} = -\beta^{-} + l_r\rho_1(t)r^{+} \end{cases} \tag{28}$$

5. EXPERIMENTAL RESULTS

The interval observers are now tested on a data set acquired using a prototype vehicle. The run was performed on at test track located in the city of Versailles-Satory (France). The track is 3.5Km length with various curve profiles allowing vehicle dynamics excitation.

Several sensors are implemented on the vehicle: The yaw rate r is measured using an inertial unit, the steering angle δ_f is measured by an absolute optical encoder while an odometer provides the vehicle longitudinal speed. Finally, a high precision Correvit sensor provide a measure of the sideslip angle. This measure is not used for observer design. It serves only for observer estimation evaluation. The steering angle and the vehicle longitudinal speed profiles are shown in figures 3 and 4. One can see that the speed should be treated as a time-varying parameter.

In addition, on can see from these figures that the steering angle at the tire level reaches 0.1 rad while the speed is about 14 m/s. The corresponding lateral acceleration is about 4.2 m/s^2. The lateral forces reach thus the nonlinear zone. Finally, for our purpose, we assume that the cornering stiffness parameters are affected of 10% uncertainty in their value.

The results for the LPV interval observer (18) are shown in Fig. 6, 7, 8 and 9, the interval observer provides the guaranteed bounds covering the trajectory of state variables. The algebraically reconstructed lateral forces fulfill the interval requirements. During the maneuver, both the front and the rear tire forces saturate. One can see on figure 10 that the real front tire force is within the envelope defined by the interval observer both in the linear and the nonlinear region.

The initial conditions are chosen different from that of the measurements. The convergence time is short and the intervals width are tight. In figure 11, the interval errors $e_\beta = \beta^{+} - \beta^{-}$ and $e_r = r^{+} - r-$ are shown. We note that the interval width is related to the model (8) uncertainty. If the corning stiffness parameters are perfectly known and the model does not contain uncertainties therefore estimated bounds will converge asymptotically to the real state.

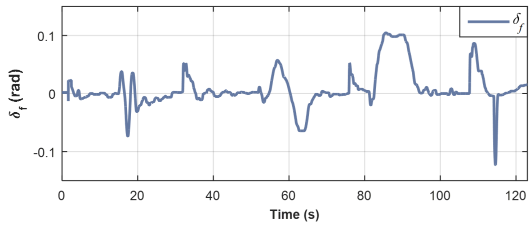

Fig. 4. Steering angle.

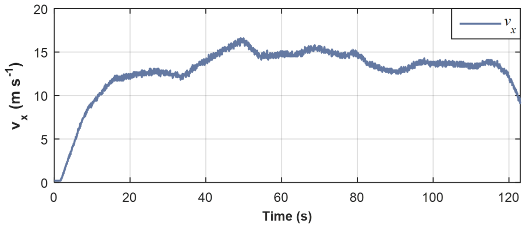

Fig. 5. Longitudinal velocity.

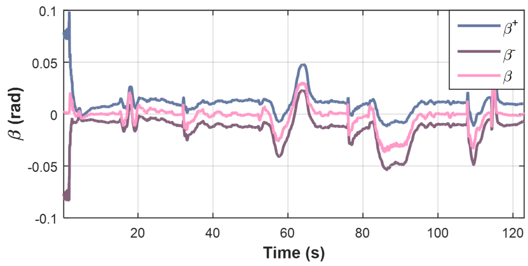

Fig. 6. Interval estimation for sideslip angle.

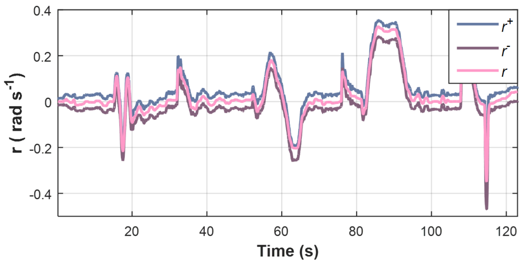

Fig. 7. Interval estimation for yaw rate.

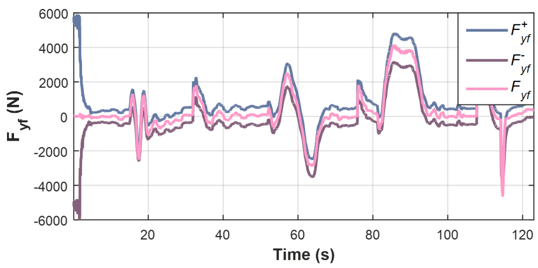

Fig. 8. Interval estimation for the front lateral tire force.

6. CONCLUSION

In this work, it has been shown how one can use interval observers for a robust estimation of sideslip angle

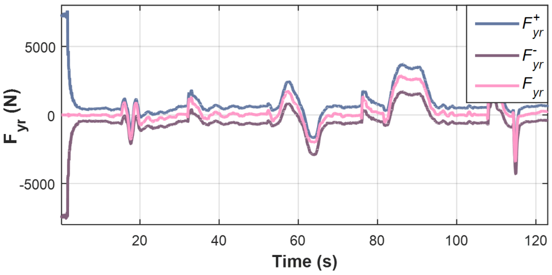

Fig. 9. Interval estimation for the rear lateral tire force.

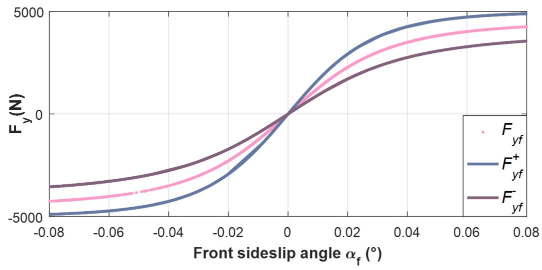

Fig. 10. Interval observer of the real front tire force.

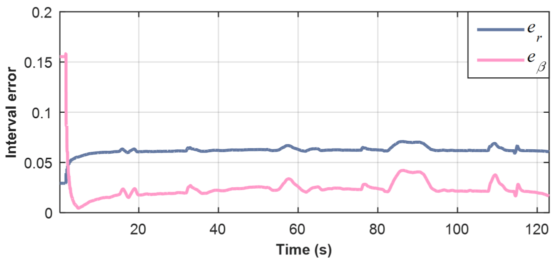

Fig. 11. Interval errors e_β and e_r.

and lateral tire forces form a two-wheeled vehicle model subject to interval uncertainties (cornering stiffness). The longitudinal velocity is treated as the online measurable time-varying parameter, the proposed interval observer is time-varying in respect to $\rho(t)$. The simulation results demonstrate the validity of proposed approach.

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